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Parametric and Harmonic Instability Interactions

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Summary

The nonlinear dynamic stability of structure due to vertical and horizontal ground motions is investigated in this paper. The structure is ideally shown as a plane frame, and the ground motion activity is described with vertical and horizontal components of the ground acceleration. The regions of unstable oscillations due to horizontal and vertical excitation are determined numerically. The results of some particular solutions are shown in Poincaré maps. They reveal periodic, quasi-periodic and chaotic vibrations.

Introduction

The design of building structures is generally carried out within the framework of static stability analysis, to assure stability. This paper deals with simple structure type in which the parametric load arises from the inertia force due to the ground motion. Earthquake activity is described with two components of the ground acceleration. The horizontal component of the acceleration is the primary cause of the vibrations in the structure calculus. The vertical ground motions component, although often disregarded in the calculations, has two separate effects: it causes axial vibrations (in the column axis direction) and, at the same time, has a parametric influence on lateral vibrations. Parametric influence depends on the earthquake activity conditions and on the characteristics of the given structure. This influence, in specific conditions, becomes dominant - it can cause parametric resonance and jeopardize the structure stability. In the frame structures, the first axial vibration frequency is much higher than the first lateral vibration frequency (bending of the columns). Because of this we can separate the instability analysis of the lateral vibrations caused by parametric resonance in the main area from the vertical (axial) vibrations.

Model Description

The structure is modelled as a plane frame (Fig. 1). Frame columns are absolutely elastic, have the length h, constant bending stiffness EI, constant distributed mass m, and are connected with an absolutely stiff beam whose total mass is $M = 2hm$. The lateral deflection of the column is described with the function $v(y,t)$, where y is the coordinate on the column axis.

$$v(y,t) = \sum_{i=1}^{n} q_i(t)\,\varphi_i(y) \tag{1}$$

The model vibration equations are derived using Lagrange's equations. The shape functions $\varphi_i(y)$ were derived from a solution of model free vibration analysis. The uniform mass

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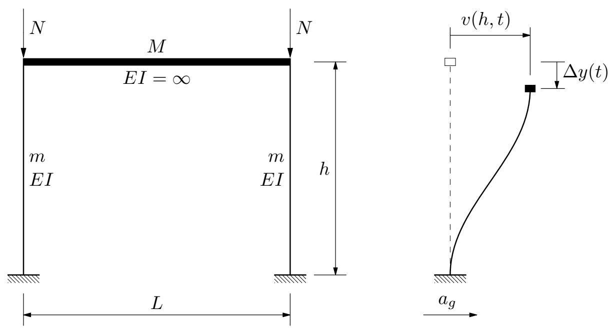

Figure 1: Frame model

column with rigid lumped beam mass on the top was considered, as shown in Fig. 1. The boundary conditions to be satisfied are:

$$
\begin{aligned}
&\varphi_i(0) = 0, \qquad && \varphi_i'(0) = 0, \\
&\varphi_i'(h) = 0, \qquad && 2EI\varphi_i'''(h) + \omega^2 M\varphi_i(h) = 0
\end{aligned} \tag{2}
$$

The free lateral frame vibration mode shapes and frequencies are determined analytically, with respect of the border conditions and the assumed beam mass:

$$
\begin{aligned}
\varphi_1(y) &= -1.06206\cos\left(\frac{1.71888y}{h}\right) + 1.06206\cosh\left(\frac{1.71888y}{h}\right) + \\
&\quad +1.29446\sin\left(\frac{1.71888y}{h}\right) - 1.29446\sinh\left(\frac{1.71888y}{h}\right) \\
\varphi_2(y) &= -0.63923\cos\left(\frac{4.89277y}{h}\right) + 0.63923\cosh\left(\frac{4.89277y}{h}\right) + \\
&\quad +0.63143\sin\left(\frac{4.89277y}{h}\right) - 0.63143\sinh\left(\frac{4.89277y}{h}\right) \\
\varphi_3(y) &= -0.66230\cos\left(\frac{7.96446y}{h}\right) + 0.66230\cosh\left(\frac{7.96446y}{h}\right) + \\
&\quad +0.66271\sin\left(\frac{7.96446y}{h}\right) - 0.66271\sinh\left(\frac{7.96446y}{h}\right)
\end{aligned} \tag{3}
$$

The vertical component of the ground motion is described by an ideally harmonic force $N(t)$ and is added to the constant longitudinal force P_0 (which is caused by static load). The

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horizontal component of the earthquake is described with a similar harmonic force $H(t)$.

$$\begin{aligned} N(t) &= P_0 + P_t \cos(2\psi t) \\ H(t) &= \overline{A}\cos(\beta t) \end{aligned} \tag{4}$$

The beam mass vibrates vertically due to the column deflections. This movement generates an additional inertia force, which is a second order value and depends on the frequency and amplitude of the lateral vibrations. It is clear that this force has a periodic longitudinal effect, e.g., a parametric influence on the lateral deflections. Further, this means that this influence can cause self-induced parametric resonance.

$$\Delta y = h - \int_0^h \sqrt{1 - (\frac{\partial v(y,t)}{\partial y})^2} dy = \int_0^h \frac{1}{2}(\frac{\partial v(y,t)}{\partial y})^2 dy \tag{5}$$

The mechanical system kinetic energy T consists of the column lateral vibration kinetic energy and the kinetic energy of the beam mass M. The linear influence of lateral deflection of the mass M and the non-linear vertical deflection are included.

$$T = 2\int_0^h \frac{1}{2} m(\frac{\partial v(y,t)}{\partial t})^2 dy + \frac{1}{2} M(\frac{\partial v(h,t)}{\partial t})^2 + \frac{1}{2} M(\frac{\partial \Delta y}{\partial t})^2 \tag{6}$$

The potential energy V is given as the flexural strain energy in the column.

$$V = 2\int_0^h \frac{1}{2} EI(\frac{\partial^2 v(y,t)}{\partial y^2})^2 dy \tag{7}$$

Generalized forces caused by the vertical forces $N(t)$ and horizontal force $H(t)$ are:

$$Q_i = -N(t)\frac{\partial \Delta y}{\partial q_i} + H(t)\frac{\partial v(h,t)}{\partial q_i} \tag{8}$$

After substituting equation (1) into the equations (5) - (8) and then into Lagrange's equations n differential equations of the model motion are determined:

$$\frac{1}{\omega_i}\ddot{q}_i - N(t)\sum_{j=1}^{n} q_j(t)\alpha_{i,j} + \sum_{j=1}^{n} q_j(t)\alpha_{i,j}\Gamma(q,\dot{q},\ddot{q}) = A\gamma_i cos(\beta t), \quad (i = 1,2,\ldots,n) \tag{9}$$

In the equations (9) ω_i are the free vibration frequencies, α_i and γ_i are the constants, and Γ is a function.

$$\alpha_{i,j} = \frac{\int_0^h \varphi_j' \varphi_i' dy}{EI\int_0^h \varphi_i''^2 dy} \tag{10}$$

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$$\gamma_i = \frac{h\varphi_i(h) + \int_0^h \varphi_i(y)dy}{\int_0^h \varphi_i''^2 dy} \tag{11}$$

$$\Gamma(q,\dot{q},\ddot{q}) = \sum_{p=1}^{n}\sum_{k=1}^{n}\int_0^h \varphi_p' \varphi_k' dy\,(\dot{q}_p\dot{q}_k + q_p\ddot{q}_k) \tag{12}$$

The coordinates q_i in the equations (9) are re–scaled into non-dimensional variables u_i using the column length h, and time t is re-scaled using the fundamental model frequency ω_1. The re-scaled frequencies ξ of the force H, and θ of the force N and two non-dimensional parameters μ and Ω are introduced. The parameter μ is related to the P_t and the critical axial force P_c, and the parameter Ω is the re-scaled fundamental free vibration frequency reduced by the influence of the constant axial force P_0.

$$\begin{aligned} \Omega &= \sqrt{1-\frac{P_0}{P_c}} \\ \mu &= \frac{P_t}{2(P_c-P_0)} \end{aligned} \tag{13}$$

By setting

$$\begin{aligned} u_i &= u_i, \quad \text{for} \quad i = 1,\cdots,n \\ u_i &= \dot{u}_{i-n} \quad \text{for} \quad i = (1+n),\cdots,2n, \end{aligned} \tag{14}$$

the system of n equations (9) can be replaced with the system of the $2n$ equations.

$$\dot{\mathbf{u}} = f(\mathbf{u},\dot{\mathbf{u}},\mu,\xi,\theta,A) \tag{15}$$

The obtained system of non-linear differential equations for selected small values of $P_0 = 0.2P_c$ has been solved numerically.

Unstable Regions

The solution unstable areas (for certain parameter values) are investigated in a Liapunov sense, by checking the energy increase over time. The numerical integration of the differential equation systems was repeated for each successively determined group of parameters. For the first parameter group, the areas of unstable forced lateral vibrations and parametric vertical vibrations were investigated separately. The parametric unstable regions are shown on Fig. 2 a) and the lateral force unstable regions are shown on Fig. 2 b). The main linear unstable region and a secondary, non-linear area caused by a higher value of the horizontal force amplitude A are shown on Fig. 2 b. Poincaré maps of the responses within the nonlinear unstable parameter area are shown on Fig. 2 c) and Fig. 2 d). The

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values x and v_x on the map's axis correspond to the displacement $v(h,t)$ and $\dot{v}(h,t)$ respectively. The map shows chaotic response for higher value of the horizontal force amplitude A.

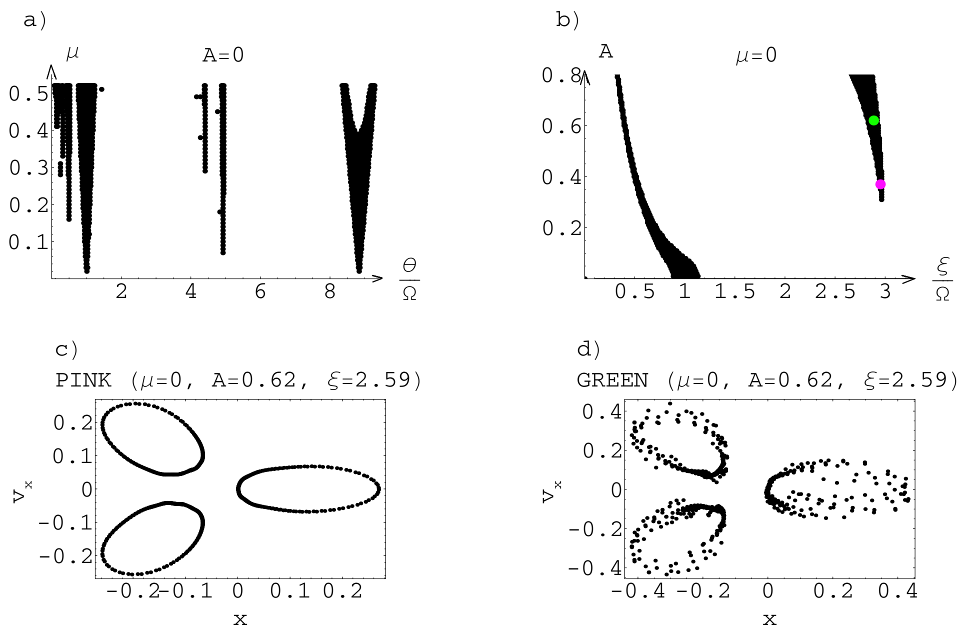

Figure 2: Unstable regions and Poincaré maps

The interactions between the forced lateral vibration and vertical force, are investigated for the different values of the force amplitude H and for the different parameter values μ. The unstable response areas due to this interactions, for the low horizontal force amplitude, and two values of the parameter μ are shown on the Fig. 3).

The areas of harmonic resonance (near $\xi = \Omega$) can be seen in both cases. The parametric resonance for the low value of P_t is visible for the main area (Fig. 3 a). For the higher value of μ, the parametric resonance is visible for both the main and the secondary area (Fig. 3 b). Both pictures show the unstable regions for the simultaneously applied horizontal and vertical forces when the vertical force frequency is twice as high as the horizontal force frequency. These regions are caused by the parametric instability. The force frequency ratios related with the other shown unstable response areas, result with the unstable sub–harmonic and super–harmonic vibrations. Poincaré maps for the two marked solutions are shown on Figure 3 c) and d).

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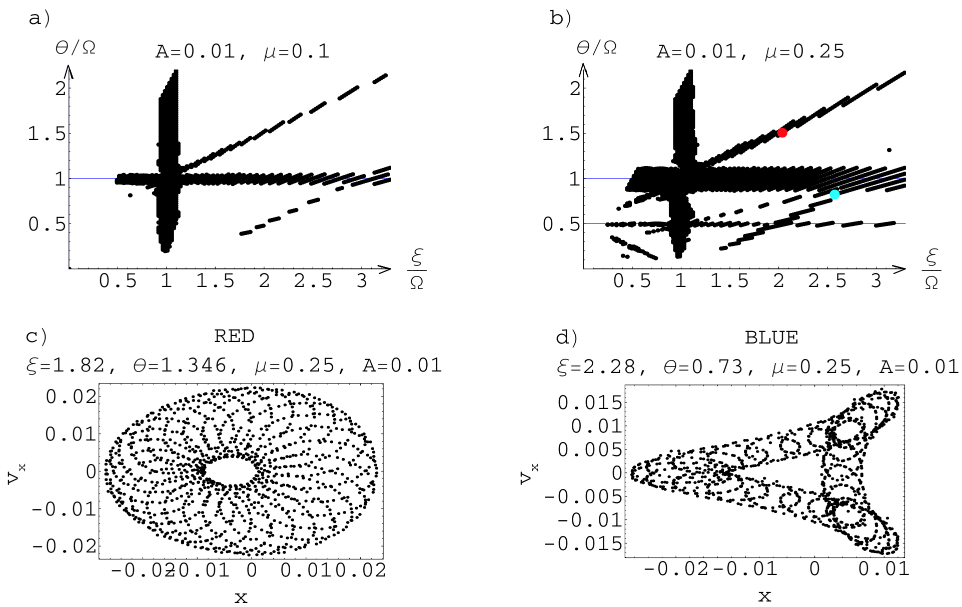

Figure 3: The interaction of horizontal and vertical force

Conclusion

The dynamic stability problems of a real structure subjected to ground motions is complicated by the fact that real structures are too complex and usually designed to respond inelastically during the earthquake. This paper shows that the effect of the interaction between vertical and lateral excitation can, for some parameter values, be significant. The results indicate that the further investigations should be made for the inelastic model with non-linear deformations.

Reference

1. Bolotin, V. V. (1964): *Dynamic Stability of Elastic Systems*, Holden-Day, Inc.
2. Hjelmstad, K. D., Williamson, E. B.: "Dynamic Stability of Structural Systems Subjected to Base Excitation", *Engineering Structures*, Vol. 20, pp. 425-432.
3. Moon, F. C. (1992): *Chaotic and Fractal Dynamics - An Introduction for Applied Scientists and Engineers*, John Wiley Sons Inc., New York
4. HaQuang, N.,Mook, D. T.,Plaut, R.H. (1897)"A non-linear Analysis of the Interaction Between Parametric and External excitations",*Journal of Sound and Vibration*,Vol.118(3), pp.425-439