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A Conceptual Structural Health Monitoring System for Fatigue Damage*

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Summary

The concept of a Structural Health Monitoring System is discussed. The system includes a permanently installed structural health-monitoring network of ultrasonic sensors. The sensors continuously provide information to a probabilistic fatigue damage analysis procedure for real time probabilistic forecasting of the remaining lifetime of a component. Here we report preliminary laboratory results. To quantify pre-cracking fatigue damage, a narrowband SAW generator and a harmonically- matched SAW receiver are used to monitor the variations in the harmonic SAW signal as a function of loading and number of cycles. The sensor data is then used in a probabilistic fatigue damage analysis. Probabilistic fatigue lives are evaluated using the Monte Carlo Method with Importance Sampling. Numerical results on the probabilistic assessment of fatigue damage are presented.

Introduction

Conventional procedures for life prediction of rotorcraft drive train components subjected to fatigue are generally based on the 'safe-life' approach (see e.g. [1]), coupled with Palmgren [2] and Miner [3] rules of linear cumulative damage. In the 'safe-life' approach for metal fatigue, life prediction is based on data from fatigue testing of components. All components of a structure are replaced when the probability of failure reaches a prescribed (often small) value, even though some of them may have a significant remaining life. Hence it is a conservative approach with an economic penalty. To avoid the penalty incurred when using the 'safe-life' approach, the 'damage-tolerant' approach is generally considered for life predictions. This approach is especially useful when the rate of damage is well understood and can be monitored with a technique of quantitative non-destructive evaluation. However, in materials such as high strength steels, critical damage in the form of a crack of detectable but very small length, often occurs late in the lifetime of a component. When a detectable crack has developed out of microscopic damage processes, it grows to an unacceptable length in a time that is short as compared to the total lifetime of the component. Continuous monitoring by a condition monitoring system can, however, significantly improve the reliability of the damage tolerance approach, and it can unite SHM and damage tolerance. Particularly if pre-crack damage can be monitored and related to crack formation by an analytical fatigue damage procedure, very substantial safety benefits can be gained.

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A systematic approach to continuous damage detection and its incorporation in a probabilistic fatigue damage analysis is illustrated in Fig. (1). As seen from Fig. (1), the structural

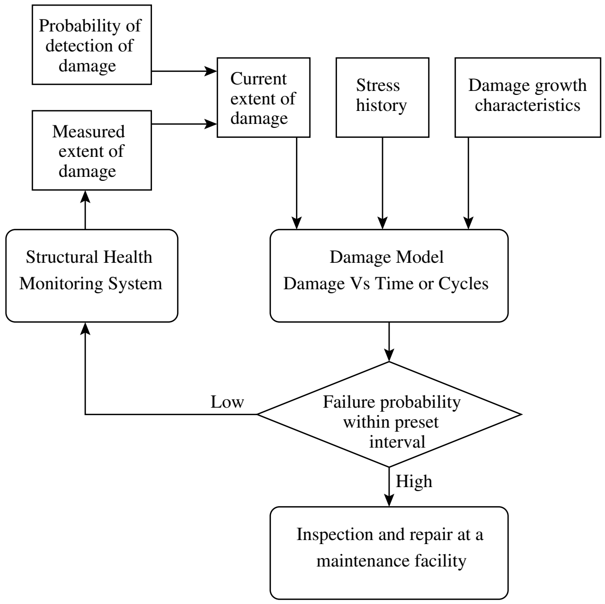

Figure 1: Continuous Lifetime Diagnostic System

health monitoring system measures the damage in the component using a NDE technique. The measured damage along with the probability of detection of the system, the stress level and the damage growth characteristics are incorporated in an appropriately chosen damage model. The probability distribution of the number of cycles to failure is then calculated from the damage model. If the probability of failure within a preset interval is high, the component is sent to a maintenance facility for a detailed inspection.

Acoustic Nonlinearity

To quantify the damage accumulation taking place in a component undergoing fatigue, it is first necessary to relate the accumulated damage to an observable variable. The accumulated damage leads to changes in the microstructure of the component which in turn leads to changes in the ultrasonic wave propagation through the specimen. The acoustic nonlinearity A_2/A_1, defined as the ratio of the second harmonic amplitude A_2 to the fundamental amplitude A_1, quantifies the extent to which an ultrasonic wave is distorted as it propagates through a nonlinear material (see e.g. [4]). Ogi et. al [5] have observed that the acoustic nonlinearity increases nearly monotonically, and shows a distinct peak at the point of macrocrack initiation. Similar increases in the acoustic nonlinearity have also been observed in preliminary tests done in our laboratory for Aluminum specimens. A 5MHz PZT transducer is used to generate the fundamental surface acoustic wave and the second harmonic is detected using a 10MHz PZT transducer. It has been found that for a fixed A_1, the amplitude A_2 increases with increasing cycles, reaches a maximum, and then decreases.

The above experiments suggest that the state of damage in a fatigue specimen can be quantified by expressing it as a function of the acoustic nonlinearity.

Damage Model

This section presents a damage model whose evolution is similar to the evolution of the nonlinearity up to the point of macrocrack initiation. The state of damage in a specimen at a particular cycle during fatigue is represented by a scalar damage function $D(N)$. The magnitude $D = 0$ corresponds to no damage, and $D = 1$ corresponds to the appearance of the first macrocrack. The following phenomenological model (see [6]) is assumed to represent the evolution of the damage

$$\frac{dD}{dN} = \begin{cases} \frac{1}{N_c}\left(\frac{\Delta\sigma/2 - r_c(\bar{\sigma})}{r_c(\bar{\sigma})}\right)^m \frac{1}{(1-D)^n} & \text{if} \quad \Delta\sigma/2 > r_c(\bar{\sigma}) \\ 0 & \text{if} \quad \Delta\sigma/2 < r_c(\bar{\sigma}) \end{cases} \tag{1}$$

Here, N_c is a normalizing constant, $\Delta\sigma$ is the stress range in a cycle, $r_c(\bar{\sigma})$ is the endurance limit when the mean stress in a cycle is $\bar{\sigma}$, and $m > 0$ and $n > 0$ are parameters which depend on the material and the loading conditions and are calculated by correlating the evolution of the nonlinearity A_2/A_1 to the evolution of D. It is assumed that $r_c(\bar{\sigma})$ follows the Goodman relation (see [7]), i.e.

$$r_c(\bar{\sigma}) = r_c(0)\left(1 - \frac{\bar{\sigma}}{\sigma_{ult}}\right)$$

where, σ_{ult} is the ultimate tensile strength of the material. Assuming that $\Delta\sigma$ and $\bar{\sigma}$ are constant during cycling and that $\Delta\sigma/2$ is always greater than $r_c(\bar{\sigma})$, Eq. (1) can be solved to obtain

$$D(N) = 1 - \left[(1-D_0)^{n+1} - \frac{N}{N_c}\left(\frac{\Delta\sigma/2 - r_c(\bar{\sigma})}{r_c(\bar{\sigma})}\right)^m (n+1)\right]^{\frac{1}{n+1}} \tag{2}$$

Here D_0 is the initial damage present in the specimen. To find the number of cycles needed for macrocrack initiation, $D = 1$ is substituted in Eq. (2) to obtain

$$N_{ini} = \frac{N_c}{n+1}(1-D_0)^{n+1}\left(\frac{r_c(\bar{\sigma})}{\Delta\sigma/2 - r_c(\bar{\sigma})}\right)^m \tag{3}$$

Probability of Macrocrack Initiation

This section describes the procedure for calculation of the probability of macrocrack initiation and is based on the model described in the previous section. Depending on the problem under consideration, the quantities appearing in Eq. (3) are suitably randomized. Let $\mathbf{X} = [X_1 \;\; X_2 \ldots X_k]^T$ denote the random quantities (for example, for a problem with known constant stress cycles, the quantities $r_c(\bar{\sigma})$, D_0, m and n can be considered random with known probability distribution and $\mathbf{X} = [X_1 \;\; X_2 \;\; X_3 \;\; X_4]^T = [r_c(\bar{\sigma}) \;\; m \;\; n \;\; D_0]^T$). Let $f_{\mathbf{X}}(\mathbf{x})$ denote the joint probability distribution of $\mathbf{X}$. To determine the probability of macrocrack initiation P_{ma}, i.e. the probability that the number of cycles to macrocrack initiation, N_{ini}, will be less than a specified number of cycles N_s, one first defines a limit state surface given by

$$g = N_{ini} - N_s$$

To account for the inspection process, let $Ninsp$ denote the cycle number at which an inspection is carried out and let $Dinsp$ denote the damage at that cycle. If no macrocrack is observed at $Ninsp$ then it follows that $Dinsp < 1$. To account for the inherent scatter in the damage measurements, the following inequality

$$Dinsp < Dactual < 1 \tag{4}$$

is assumed, where $Dactual$ is the actual damage in the specimen at $Ninsp$. Note that this is a conservative approach and it is possible that $Dactual < Dinsp$. Also note that since it is assumed that the model represents the evolution of the damage exactly, one has $D(Ninsp) = Dactual$, where $D(Ninsp)$ is the damage predicted by the model (see Eq. (2)) at $N = Ninsp$. Therefore the inequality in Eq. (4) can be replaced by

$$Dinsp < D(Ninsp) < 1 \tag{5}$$

Let E denote the event $Dinsp < D(Ninsp) < 1$. Then the probability of macrocrack initiation P_{ma}, taking into account the inspection at $Ninsp$, is given by

$$P_{ma} \equiv Pr(N_{ini} < N_s|E) = \frac{Pr((N_{ini} < N_s) \cap E)}{Pr(E)} \tag{6}$$

To calculate this probability, the two probabilities occurring on the right hand side of Eq. (6) are evaluated separately. To do this, it is first necessary to represent the event E in the space of random variables. This is achieved by defining a function $h(\mathbf{x})$, such that

$$h(\mathbf{x}) = \left[(1 - D_0)^{n+1} - \frac{Ninsp}{N_c}\left(\frac{\Delta\sigma/2 - r_c(\bar{\sigma})}{r_c(\bar{\sigma})}\right)^m (n+1)\right]^{\frac{1}{n+1}}$$

From Eq. (2) it follows that Eq. (5) is equivalent to $Dinsp < 1 - h(\mathbf{x}) < 1$. This statement is equivalent to

$$0 < h(\mathbf{x}) < 1 - Dinsp$$

which represents the event E in the space of random variables. Eq. (2) shows that at $N = Ninsp$, the surface $h(\mathbf{x}) = 0$ corresponds to $D(Ninsp) = 1$, and the surface $h(\mathbf{x}) = 1 - Dinsp$ corresponds to $D(Ninsp) = Dinsp$. The probability of the event E is now given by

$$Pr(E) = \int\limits_{0<h(\mathbf{x})<1-Dinsp} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x} \tag{7}$$

and

$$Pr((N_{ini} < N_s) \cap E) = \int\limits_{(g(\mathbf{x})<0)\cap(0<h(\mathbf{x})<1-Dinsp)} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x} \tag{8}$$

To evaluate the integrals appearing in Eqs. (7) and (8), the random variables are first mapped via a Rosenblatt transformation (see [8]) into a standard Gaussian space where the random variables denoted by $\mathbf{U} = [U_1 \;\; U_2 \ldots U_k]^T$ are independent, normally distributed and have zero mean and unit standard deviation. The modified Hasofer-Lind, Rackwitz-Fiessler (HL-RF) algorithm described in [9] is used to obtain the point closest to the origin on the surface $g(\mathbf{u}) = 0$, $\mathbf{u}^*$. In the modified HL-RF algorithm, one adjusts the step size during each iteration to obtain a sufficient decrease in the merit function which is based on the first order optimality conditions. Monte Carlo integration with importance sampling, with the sampling density centered at $\mathbf{u}^*$ (see [10]) is then used to calculate the integrals in Eqs. (7) and (8).

Sample Problem

To demonstrate the application of the ideas presented here, the probability of macrocrack initiation is calculated for a sample problem. The data, i.e. the acoustic nonlinearity as a function of the number of cycles, is obtained from [5]. Ogi and his co-authors have performed a rotating bending fatigue test with a four point bending configuration on a 0.25% C (mass) steel and have obtained the variation of the acoustic nonlinearity with the number of cycles. The yield strength of the material is 333 MPa specimen and it is subjected to a maximum bending stress of 280 MPa. The endurance limit of the material at zero mean stress, $r_c(0)$, is assumed to be 180 MPa.

To simulate the inspection process, the acoustic nonlinearity is obtained from [5] at different numbers of cycles with the interval between the cycles getting progressively shorter. Table (1) shows the acoustic nonlinearity measured as a function of number of cycles. It is observed

Table 1: Measured Values of Acoustic Nonlinearity during Successive Inspections

j	$Ninsp_j$	$(A_2/A_1)_j \times 10^{-3}$
0	0	0.90
1	11200	0.80
2	22400	0.90
3	26880	1.50
4	30800	2.00
5	33040	2.50
6	34000	3.10

Table 2: 'Measured' Values of Damage during Successive Inspections

j	$Ninsp_j$	$Dinsp_j$
0	0	0
1	11200	0.2462
2	22400	0.2769
3	26880	0.4615
4	30800	0.6154
5	33040	0.7692
6	34000	0.9539

that the evolution of the acoustic nonlinearity is not strictly monotonic during the initial stages of fatigue. The damage is obtained by normalizing the nonlinearity measurements by the expected maximum value of the nonlinearity. For the given problem the maximum value is assumed to be 3.25×10^{-3}. It is also assumed that the specimen is initially undamaged, i.e. $Dinsp_0 = 0$. The 'measured' damage which is calculated from the corresponding nonlinearity measurements is tabulated in Table (2). Using the values of damage given in Table (2), the constants m and n are calculated using nonlinear regression (e.g. see [11]). The values are calculated starting from the third inspection. It is assumed that the damage values from $0 \ldots j$ inspections are available to calculate m and n at the j inspection. The probability of macrocrack initiation is then calculated by assuming that $\Delta\sigma$, $\bar{\sigma}$, N_c and D_0 are fixed quantities while $r_c(0)$, m and n are independent random variables each having a lognormal distribution. The following values are used for the fixed quantities: $\Delta\sigma = 2\times$ 280 MPa, $\bar{\sigma} = 0$ MPa, $N_c =$ 10000, $r_c(0) = 180$ MPa and $D_0 = 0$. The random variable $r_c(0)$ is assumed have a mean of 180 MPa with a standard deviation of 5.4 MPa. The probability of macrocrack initiation is calculated as described in Section after each inspection for different N_s and is given in Table (3).

Note that for the fatigue problem described, the first macrocrack is observed at approximately 34160 cycles (see [5]). As seen from Table (3), the formation of the macrocrack is predicted quite well in spite of using a simple damage model and making simple assumptions regarding the parameters involved in the model.

Table 3: Calculation of P_{ma}

Cycles	P_{ma}			
N_s	3rd Insp (Ninsp = 26880)	4th Insp (Ninsp = 30800)	5th Insp (Ninsp = 33040)	6th Insp (Ninsp = 34000)
30000	0.03630	0.00000	0.00000	0.00000
31000	0.05274	0.05528	0.00000	0.00000
32000	0.07142	0.31742	0.00000	0.00000
33000	0.09305	0.54063	0.00000	0.00000
34000	0.11695	0.70945	0.83558	0.00000
35000	0.14302	0.82748	0.992680	1.00000
40000	0.30510	0.99570	1.00000	1.00000

Conclusions

Acoustic nonlinearity has been used to quantify the fatigue damage for a damage model which is coupled with reliability analysis. Probabilistic fatigue lives are evaluated for a sample problem.

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