Advances in Computational & Experimental Engineering & Science

Energetic-Statistical Size Effect in Composites and Sandwich Structures

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Summary

While the lecture reviews the recent advances at Northwestern University in the prediction of size effects in fiber-polymer composites and sandwich structures made of such composites and a foam core, the present brief article summarizes only one part—a novel microplane-type material model for fiber composite laminates that can capture the postpeak softening damage, which is necessary for the numerical simulation of size effect in such laminates. Detailed presentation of this model is planned for a journal article.

Advances in Modeling of Energetic-Statistical Size Effect in Composites

The conference lecture gives an overview of the problems of scaling and size effect in structural failure, which have not come to the forefront of attention until closing years of the last century. The problem is important for large ship structures made of composites, and especially for predicting the energy absorption capability under dynamic loads.

The classical view that any observed size effect was statistical was reversed during the 1980s, and for composites the 1990s. As is now widely accepted, quasibrittle materials including concrete, rock, tough ceramics, sea ice, snow slabs and composites exhibit major size effects on the mean structural strength that are deterministic in nature, being caused by stress redistribution and energy release associated with stable propagation of large fractures or with formation of large zones of distributed cracking.

The lecture begins by reviewing the general asymptotic properties of size effect implied by the cohesive crack model or crack band model, and highlights the use of asymptotic matching techniques as a means of obtaining scale-bridging size effect laws representing a smooth transition between two power laws. Attention is focused on size effects observed in fiber-polymer

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composites failing either by tensile fracture or by propagation of compression kink bands with fiber microbuckling. The size effects in polymeric foams and sandwich structures are also discussed. A nonlocal model for incorporating the Weibull-type statistical size effect due to local strength randomness into the energetic size effect theory is outlined next, and the predictions of the combined nonlocal energetic statistical theory are compared to experimental evidence. The probabilistic-energetic modeling exploits the stability postulate of extreme value statistics.

Finally, a novel microplane-type material model for fiber composite laminates that can capture the postpeak softening damage is outlined. This model (the detailed presentation of which is planned for a journal article) is needed for a realistic finite element simulation of size effect in such laminates. Due to scope limitations, the subsequent compact exposition[3] deals exclusively with this new model.

Microplane Model for Laminates with Quasi-Brittle Matrix

While general constitutive models for complex three-dimensional elasto-plastic behavior and fracturing damage exist for metals, soils, concrete, etc., they are still unavailable for fiber composites. Progress in design necessitates the development of such a model for composites. Therefore, a systematic effort in this direction has been initiated.

The microplane model, which began by Bažant's modification of the classical Taylor models, has been developed for concrete, sea ice, rock, soils and polymeric foam. In this model, the tensorial invariance restrictions and the use of tensors and their invariants are bypassed by formulating the constitutive law as a relation between the stress and strain vectors on general planes in the material, called the microplanes. The responses from the microplanes of all possible orientations are then combined according to a variational principle to obtain the response of the macroscopic continuum. In the case of softening damage, this must be done under a kinematic (rather than static) constraint, in which it is assumed that the strain vectors on all the microplanes are the projections of the macroscopic strain tensor.

This approach is computationally more demanding than the classical tensorial approach, but this no longer matters for the state-of-art powerful computers. The benefit is conceptual simplicity and the ability of the

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microplane constitutive law to simulate directly various oriented physical phenomena, such as oriented frictional slip, oriented crack opening, transverse expansion, etc.

Background: This paper focuses on quasi-brittle matrix composites. Therefore, the microplane model formulation applied for the matrix component is similar to that developed for concrete [2, 3] and its basic concepts will now be briefly reviewed.

In the kinematically constrained microplane model, the strain tensor is projected on the microplanes of all orientations $\mathbf{n} = [n_1, n_2, n_3]^T$, defined on the basis of an optimal integration formula [1], as follows:

$$\epsilon_N = N_{ij}\epsilon_{ij},\ \epsilon_M = M_{ij}\epsilon_{ij},\ \epsilon_L = L_{ij}\epsilon_{ij} \tag{1}$$

where ϵ_N is the normal strain; ϵ_L and ϵ_M are tangential strain; $N_{ij} = n_i n_j$ $(i,j = 1,2,3)$, $L_{ij} = (n_i l_j + n_j l_i)/2$, $M_{ij} = (n_i m_j + n_j m_i)/2$ with $\mathbf{l} = [l_1, l_2, l_3]^T$ and $\mathbf{m} = [m_1, m_2, m_3]^T$. Vectors $\mathbf{n}$, $\mathbf{l}$ and $\mathbf{m}$ represent an orthogonal system in the local coordinates of the microplane. The stresses are computed from the strains through vectorial constitutive laws on the basis of the following algorithm:

1) Compute the elastic stress predictor:

$$\sigma_V = E_V\epsilon_V,\ \sigma_D = E_D\epsilon_D,\ \sigma_N = \sigma_V + \sigma_D,\ \sigma_T = E_D\epsilon_T \tag{2}$$

where $\epsilon_V = \epsilon_{kk}/3$ is the volumetric strain, $\epsilon_D = \epsilon_N - \epsilon_V$ is the spreading strain, $\epsilon_T = \sqrt{\epsilon_L^2 + \epsilon_M^2}$ is the tangential strain; $E_V = E/(1-2\nu)$ is the volumetric modulus, $E_D = E/(1+\nu)$ is the deviatoric modulus; σ_V is the volumetric stress, σ_D is the deviatoric stress, σ_N is the normal stress, σ_T is the tangential stress. Note that E and ν are elastic Young's modulus and Poisson ratio for the isotropic material analyzed (in our case the matrix).

2) Verify boundary surfaces:

$$\sigma_V = \sigma_V^b(\epsilon_V),\ \sigma_D = \sigma_D^b(\epsilon_D),\ \sigma_N = \sigma_N^b(\epsilon_N),\ \sigma_T = \sigma_T^b(\sigma_N) \tag{3}$$

If the elastic stress predictor exceeds the boundary surface, a return on the surface for the stress is enforced (similar to traditional return mapping algorithms).

3) Compute macro-stress by the principle of virtual work:

$$\sigma_{ij} = \frac{3}{2\pi}\int_\Omega (\sigma_N N_{ij} + \sigma_M M_{ij} + \sigma_L L_{ij})d\Omega \tag{4}$$

where Ω is the unit hemi-sphere on which the integration is carried out [1].

This computational algorithm involves several parameters, which must be calibrated by experiments corresponding to different loading paths. In the case of concrete, several results are already available in the literature for this purpose, but in the case of laminates more experiments must be conducted. In particular, experimental investigation of the postpeak softening behavior of uni-directional lamina is currently conducted for uni-axial tension in the longitudinal and transverse direction with respect to the fiber.

Numerical Algorithm for Incremental Loading: On the basis of the numerical model proposed in [4], the fiber-matrix system is analyzed through a cell model. The strain components ϵ_{ij} applied to the representative volume element are distributed among the three components A_1 (fiber), A_2 (matrix on the top of the fiber) and B (matrix on the side of the fiber) in order to enforce at the matrix-fiber interface a series-parallel coupling (for which an improvement utilizing the Mori-Tanaka approach is currently investigated).

It is assumed that the inelastic phenomena are mainly related to the component B (e.g.: softening due to cracking under uniaxial transverse load), while A_1 and A_2 are elastic. Therefore, in the numerical algorithm, the elastic moduli $\mathbf{C}^{A_1} = [C^{A_1}_{ijkl}]$ of A_1 and $\mathbf{C}^{A_2} = [C^{A_2}_{ijkl}]$ of A_2 are properly combined according to a series-parallel model and the resulting stiffness tensor is called C^{A}_{ijkl}. The elastic moduli of component A_1 consider the moduli reduction caused by fiber waviness, important for fabric fiber composites or laminates with imperfections, by averaging the rotated (orthotropic) stiffness tensors of some chosen directions (microplanes) as follows:

$$\mathbf{C}^{A_1} = \frac{1}{n}\sum_{k=1}^{n} \mathbf{T}_k^{-1}\mathbf{C}\mathbf{T}_k \tag{5}$$

where n is the number of directions considered, $\mathbf{T}$ is a fourth order rotation tensor and $\mathbf{C}$ is the orthotropic elastic moduli stiffness tensor.

The central part of the computational algorithm consists of the evaluation of the stresses acting on the representative volume element for the given input strains. At first, the strains are distributed between the two components A and B as:

$$\epsilon^{A}_{ij} = A^{A}_{ijkl}\epsilon_{kl} \tag{6}$$

$$\epsilon^{B}_{ij} = A^{B}_{ijkl}\epsilon_{kl} \tag{7}$$

where ϵ_{ij}^A and ϵ_{ij}^B are the strain tensors acting on component A and B respectively; A_{ijkl}^A and A_{ijkl}^B are the strain concentration tensors, defined on the basis of the series-parallel condition (and currently being improved by the Mori-Tanaka approach). Later, the stress in component A is obtained directly as $\sigma_{ij}^A = C_{ijkl}^A \epsilon_{kl}^A$, while the stress σ_{ij}^B in component B corresponding to the strain ϵ_{ij}^B is obtained by the kinematically constrained microplane model described above.

The onset of inelastic deformation is characterized by the fact that the parallel condition for the stresses is not satisfied any more at the interface. Therefore, the corresponding strain quantity must be modified on the basis of a Newton-Raphson iteration until convergence is achieved.

Finally, the stress is obtained as a weighted average of the two stress components:

$$\sigma_{ij} = \nu^A \sigma_{ij}^A + \nu^B \sigma_{ij}^B \tag{8}$$

where ν_A and ν_B represent the volume fraction of component A and B respectively.

Numerical Predictions of Elastic Moduli and Softening: The computational algorithm described is applied for the computation of the elastic moduli of a uni-directional carbon-epoxy composite with transversely orthotropic fibers properties $E_{11}^f = 276$ GPa, $E_{22}^f = 13.8$ GPa, $G_{12}^f = 20$ GPa, $\nu_{12}^f = 0.25$ and isotropic matrix properties $E^m = 3.4$ GPa, $\nu^m = 0.35$. In particular, Fig. 1 (left) shows the results for the transverse modulus E_{22} for different values of the fiber volume fraction ν_f by solid circle points. These results are compared with a formula from the literature [5] (see straight line in Fig. 1 on the left), which is seen to be in a very good agreement with the experimental data available. The agreement between the computation and the formula confirms the validity of the model.

Another example is given in Fig. 1 (right), where the line describes the result for the same carbon-epoxy composites (with $\nu_f = 0.6$) loaded under transverse uniaxial tension. The lamina behavior is matrix dominated; therefore, the overall behavior is quasibrittle. The stress and strain at peak are calibrated on the basis of the transverse strength and transverse strain at failure of the composite lamina available in the literature. However, the slope of the softening curve is still under experimental investigation.

Closing Comment: The microplane-type modeling of the progressive

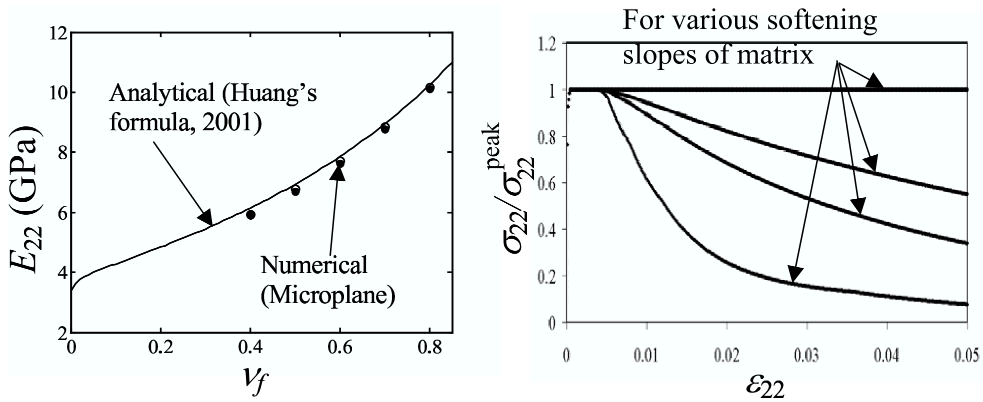

Figure 1: Microplane numerical predictions of elastic moduli (left) and transverse softening (right).

failure of laminates promises to greatly improve the predictions of size effect in static failures and the energy absorption capability of laminates and sandwiches under loading by impact, blast and shock.

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