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Optimal Dispatch of Hydro Units Considering Head Dependency

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Summary

The powerhouse input-output characteristics (I/O) of hydro power plants are major curves for the optimization problem of central exploitation [1]. Some methods had been applied for the I/O curves determination without the head loss effect consideration. This paper presents a new method that takes into account the head loss effect, and is based on optimization techniques, using Lagrangean Relaxation [2-6]. This method, due to unit's proper characteristics, requires that the cost of each unit (water flow trough powerhouse) must be expressed in terms of quadratic costs. Trough this approach, the obtained results are optimal, but referring to a convex objective function (approximated to the original function), with added advantage on reduced computation time.

Introduction

The electric utility deregulation and restructuring in Portugal has been implemented in a step-by-step way, and is now based on the existence of both Public Service Electric System (SEP) and Independent Electric System (SEI). The Non-binding Electric System (SENV) is part of SEI. The non-binding client is an individual or corporate body, the holder of an electric energy consumer installation, which has been authorized access to the SENV. The non-binding producer is the holder of a non-binding electric energy production license, by which it is authorized to carry out the activity of the production of electric energy within the ambit of the SENV. Concerning the economic relationship between SEP and SENV, these are clearly regulated, and within the SENV, they can be done by physical bilateral contracts, contracts for small time period, contracts with guaranteed delivery or trough offers system contracts [7]. By this mean, the electricity market liberalization process has introduced power generation concurrency as well as the possibility of the consumer (non-binding client) to choose which deliverer he wants (non--binding producer).

This new scenario brings new problems in electric energy management. One of these new problems is the exploitation of hydro power plants, and is within the responsibility of the non-binding producer. It can be state that the first approach for this problem solution it is the achievement of the powerhouse input-output characteristics, considering the head loss effect. In [1] a new method of implicit enumeration, for central characteristic curves determination was presented, considering all decision possibilities, and of those the best ones were chosen. By this way it is guarantee that the obtained results are optimal and globally optimal. But, the developed method presents a

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considerable computation time (up to 24 hours). This paper obviates this problem, through the application of the method proposed by the authors in [8] based on optimization techniques, using Lagrangean Relaxation. Although that method do not consider the head loss effect, and thus is not suitable for small hydro power plants. In this paper one consider the same method but with head loss effect consideration. As previously refereed, due to the unit characteristics, the unit costs are expressed in terms of quadratic cost. Trough this approach, the obtained results are optimal, but referring to a convex objective function (approximated to the original function), with added advantage on reduced computation time (seconds). The paper presents the main problem and his mathematical formulation, as well as the computational adopted method for solving it. After, some illustration results are presented and finally some conclusions are taken.

Problem Formulation

Given the imposed constraints, those required for each unit and those connected with all units, a proper unit commitment decision must be chosen and must be optimal from the economic benefit point of view. This problem involves, by one way, the statement of all possible decision and the value associated with each of them, and by another way, the strategy analysis used to achieve the optimal solution. Thus, the problem formulation brings another problem, of mathematical programming, non-linear, described as follows:

Mathematical Formulation

Consider a hydro power plant with *N* units. Each unit is characterized by three variables: power, water flow and head. If one of these variables is kept constant — let be the head — each unit *j* is characterized by a set of curves. The number of curves *I* is as big as bigger are the discretization levels, assumed for the head.

Each curve *i,* of unit *j*, with head $h_i = c^{te}$ is:

$$q_{ji} = f(p_{ji}, h_i = c^{te}) \tag{1}$$

with $i = 1, \ldots, I$ and $j = 1, \ldots, N$

being:

q_{ji} : water flow trough power house in [$m^3 s^{-1}$] of the unit *j* in curve *i*

p_{ji} : power generated in [MW] by unit *j* in curve *i*

h_i : head in curve *I*

The goodness of different possible decisions is made based on an established scale that characterizes each solution. This measurement scale is obtained from a function — objective function. The objective function that better fits the problem under analysis is the water flow measure (the water flow represents the operating cost).

Thus, expression (1) is a cost operation function, and the main problem to determinate the hydro power plant characteristic curves is related to the optimal unit commitment problem, and can be presented in such a way:

For a set of units within a hydro power plant, minimize the operating cost, according to:

- power demand — constraint connected with all units
- minimum and maximum generating capacity of each unit depending on head — constraint on individual curve
- minimum and maximum generating capacity of each independently on head — constraint on individual unit

So, the hydro unit commitment problem (**P**) can be written as:

$$(\mathbf{P}) \qquad \underset{u}{Min} \left(\sum_{j=1}^{N} q_{ji}(p_{ji}, h_i, u_j) \right) \tag{2}$$

subject to:

$$\sum_{j=1}^{n} p_{ji} = P \tag{3}$$

$$p_{ji}^{\min} < p_{ji} < p_{ji}^{\max} \;\cap\; p_j^{\min} < p_j < p_j^{\max} \tag{4}$$

where:

$$u_j \in \mathrm{U}_j \qquad j = 1, \ldots, N \tag{5}$$

Expression (2) represents the total value of water flow and indicates that for a specific value of generated power *P*, with head h_i, the water flow depends on the unit commitment. Expression (3) represents the power generated by the plant, with a number of units on. Expression (4) is the result of considering the minimum and maximum generating capacity of unit *j* in curve *i*, together with the minimum and maximum generating capacity of unit *j* whatever the curve is. In expressions (2) to (5) the

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represented variables are: N number of units; q_{ji}, p_{ji} and h_i symbols already defined in (1); u_j decision variable for unit j; P total power generated by plant; $p_{ji}^{\min}$ and $p_{ji}^{\max}$ minimum and maximum generating capacity of unit j in curve i; $p_j^{\min}$ and $p_j^{\max}$ minimum and maximum generating capacity of unit j (whatever the curve i); $u_j \in \mathsf{U}_j$ set of admissible decisions.

The objective function is non-linear and non convex. For these reasons problem (***P***) is very difficult to solve, being necessary the optimization out of conventional non-linear programming. The non-linear programming algorithms optimization make use of interpolation techniques that convert to convex the previously non-convex function. In this paper, the adopted method is based on optimization techniques using Lagrangean relaxation [2-6].

Lagrangean relaxation allows to relax the load constraint (3) that connects all units, as the same demanding load is satisfied by all operating units, being the load constraint possible to be violated. However, the relaxed constraint is not completely considered to be negligible. In fact, the weakness of problem (***P***) is linearly penalized in Lagrange function, by mean of Lagrange multiplier λ, in order to avoid the constraint violation. That function (represented by L) appears from the consideration of constraint (3) in the objective function (***P***). In such a way, the Lagrange function, resulting from problem (***P***), as stated in (2), trough constraint relaxation (3), can be written by:

$$\mathsf{L}\left(p_{ji},h_i,u_j,\lambda\right)=\sum_{j=1}^{N} q_{ji}(p_{ji},h_i,u_j)+\lambda\left(P-\sum_{j=1}^{N} p_{ji}\right) \tag{6}$$

For the characteristic curves evaluation the Lagrange function must be minimized and subjected to local constraints. The minimization problem is formulated as in (***Q***):

$$(\boldsymbol{Q}) \quad \underset{u}{Min}\ \mathsf{L}\left(p_{ji},h_i,u_j,\lambda\right) \tag{7}$$

subject to:

$$p_{ji}^{\min} < p_{ji} < p_{ji}^{\max} \ \cap \ p_j^{\min} < p_j < p_j^{\max} \tag{8}$$

where:

$$u_j \in \mathsf{U}_j \qquad j=1,\ldots,N \tag{9}$$

Once the problem ($\mathcal{Q}$) solution is obtained, for each head value and according to the decision variable u_j, the plant characteristic curves are obtained too — the optimal dispatch of hydro units considering head dependency (loss head effect) is achieved. These curves allow to know the power delivered by the plant and the available unit commitment, with minimum water flow. In such a way, the plant, as well as all his units, becomes characterized by one combination of three variables: power, water flow and head. This combination is non-linear and presents critical points, corresponding to discontinuities (generate/not generate points).

Illustration results

As an illustration of problem ($\mathcal{Q}$) solution, a small hydro power plant with six units was considered. Each unit is characterized by five curves, and the relation between heads is given by $h_{i+1} > h_i$ with $i = 1,\Lambda,5$. In [8] a similar illustration was presented without the head loss effect consideration — here assumed as h_i and is shown in Fig. 1 with dashed lines. The powerhouse input-output characteristics (I/O) of the hydro power plant are then obtained with head loss effect consideration — here assumed as $h_i^{'}$ and are shown in Fig. 1 with solid lines.

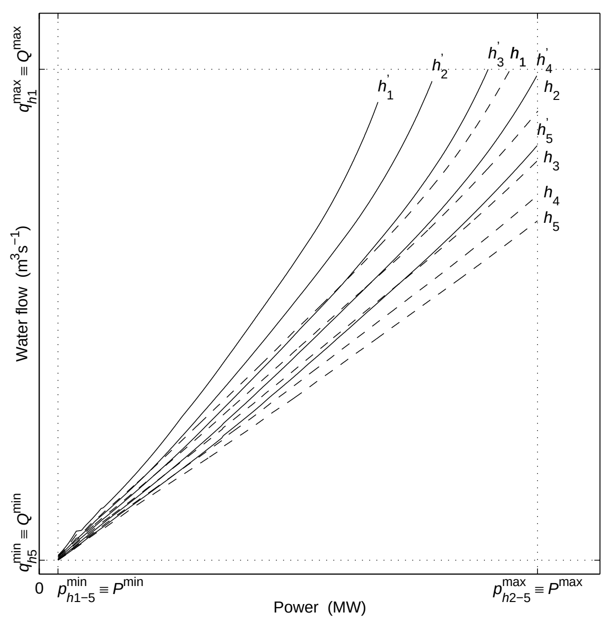

Figure 1 - Optimal dispatch of hydro units considering head dependency

Fig. 1 analysis allows concluding that the head loss effect tends to reduce the power plant efficiency. This effect is a major factor on the central exploitation optimization and must be taken into account. For small hydro power plant and for the presented illustration problem, the head loss effect reduces the power plant efficiency in about 20 per cent.

Conclusion

The optimal dispatch of hydro units considering head dependency was obtained. The dispatch solution are powerhouse input-output characteristics (I/O) of hydro power plants considering the head loss effect. This are major curves for the optimization problem of central exploitation. The optimal dispatch allowed to evaluate all possible generated power values, which units must be used, with which water flow and at what power level.

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