Homogenisation and damage mechanics for a variety of layered systems

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Introduction

A major concern for engineers is establishing that any component designed using composite materials will perform reliably and safely during service. If an engineering component becomes unserviceable during use, then it has to be regarded as having failed. For example, if a composite component breaks suddenly into two or more distinct pieces then it clearly has failed. However, if instead the component develops microstructural damage such as that associated with localised cracking in the plies of a laminate, delamination of inter-ply interfaces, fibre failure and fibre/matrix interface failure, the stiffness of the component could be degraded to the point at which it becomes unserviceable. For components such as fluid-containing tanks, pipelines and pressure vessels, if leakage occurs then the component must be regarded as having failed. For this case ply cracking in adjacent plies of a laminate is sufficient to enable leakage paths and thus failure, even though the component continues to be capable of supporting mechanical loads safely. The longer-term performance of composite components can be affected by degradation arising from exposure to aggressive environments (static fatigue), and to cyclic loads (dynamic fatigue). It is clear that failure is a concept that cannot be defined uniquely and must often be associated with the engineering purpose for which the component was designed.

Stress transfer models and energy based methods of predicting damage initiation and growth have been developed which are known to be accurate and which can be used to construct simulations of progressive damage in laminated composites [1-3]. Although primarily developed for composite materials, the models can be applied to damage development in other types of layered system, such as multi-layered coatings, layered systems arising during the steam oxidation of steels, and in super-lattices where alternating very thin layers (thicknesses measured in nanometres) of at least two different materials produce composite materials having special properties requiring the development of sound understanding. The approach of the presentation is to describe in general terms the mechanics basis of the approach to damage development in composite laminates, and to show examples of applications in the traditional composites field, and of other layered systems having significant practical importance. The approach will also draw attention to some short-comings with the use of conventional failure criteria, including those arising when applying the concepts to layered composite systems having geometries defined at the micrometre rather than millimetre scale.

Conventional failure criteria

A large number of failure criteria, designed to try to ensure that components are not subject to sudden catastrophic failure, have been proposed for composite materials, and some of these are included in commercially available finite element software systems. Invariably the engineer is offered a number of criteria that can be applied to FEA solutions, but which one should he use? The wise engineer will try them all and design on the basis of the most pessimistic prediction. This approach is not efficient, and it may not even be safe. To address this very important issue regarding the prediction of the failure of composite materials, an international Failure Prediction

Exercise has been undertaken during the past ten years or so (see Ref.[4]). It is emphasised that these criteria do not have a physical basis, and furthermore their application requires input data from uniaxial tension and compression tests, and pure shear tests. It is also emphasised that the test cases used in the Exercise did not consider any of the following effects that can be important for practical applications:

1. effects of varying the thickness of the plies, (considered in this extended Abstract)
2. effects of varying ply lay-ups in laminates,
3. effects of in-plane shear loading combined with axial and or transverse in-plane loading.

Fibre failure

For some applications, component failure requires the fracture of fibres in the composite. When an isolated fibre is tested, its strength at a given temperature can be characterised either by the stress or strain at failure. The former is a linear function of the latter. For simplified approaches for a UD composite and a cross-ply laminate, based on the parallel bar model, either strength parameter can be used if the fibres are regarded as being a loose bundle when investigating the axial behaviour of the composite. If, however, the fibres in the 0^o plies are modelled as being perfectly bonded to the matrix, then a triaxial state of stress can exist in the fibre. Assuming an axi-symmetric state of stress, as would arise when using a concentric cylinder model of a UD composite, it is clear that the transverse stress in the fibre and the thermal residual stresses can affect the axial strain in the fibre. The following question then arises: does the fibre break when a critical axial stress is reached, or a critical axial strain? It should be noted, from the general axial stress-strain relation for the fibre, that infinite stresses could in fact be applied while maintaining a bounded axial strain, provided that the ratio of the axial to the transverse fibre stresses is equal to Poisson's ratio for the fibre. This fact alone suggests that the correct failure criterion for a fibre should be based on axial fibre strain. Such an approach is reinforced if one considers a molecular model of the fibre where atoms interact through central forces that diminish to zero in magnitude as the atoms separate to infinity. While a critical fibre strain criterion has some physical justification, its use requires a careful definition of strain in relation to the temperature. For this reason it is preferable to characterise the failure of isolated fibres using strength, but to impose a critical strain criterion when such strength data is applied to fibres in a UD composite or in a laminate.

Ply crack formation

The initial damage mode for composite laminates is invariably ply cracking in off-axis plies. This damage mode is also the easiest to model reliably. Ply crack formation in laminates is, however, a complex phenomenon due to the various different locations that are available for cracking, and its dependence on ply properties and geometry such as ply thickness and ply lay-up (see Ref. [1-3]). Ply cracking is affected also by edge effects where complex stress transfer between plies is occurring at the stress-free edges of laminates. This problem is dealt with by applying models only to internal regions of the laminate for which edge effects have diminished and become negligible.

Effective stresses

The development of a model of ply cracking in a general symmetric laminate based on homogenisation principles involves the definition of effective laminate stresses that are applied to the laminate edges. The effective stresses are averages of the actual tractions that would be applied to the laminate boundaries if an exact solution for the stress and displacement distributions were to be available. The deformation applied to the laminate is in fact through external displacements that would produce a homogeneous strain field in an undamaged laminate. The corresponding applied traction distribution that would result when considering a laminate having ply cracks is complex and non-uniform. By defining the effective stresses to be averages of the actual tractions, the work

done by the averaged stresses will be identical to the work done by the actual tractions. This is a key property of the effective stresses that enables the model to develop accurate relatively simple formulae for the ply cracking stresses, and elastic constants of damaged laminates when using the effective stresses in homogenised stress-strain relations.

Thermoelastic constants

It is essential to be able to calculate the homogenised thermo-elastic properties of a general symmetric laminate in both damaged and undamaged states. In references [1-3] it was shown that ply crack development during loading (in the absence of shear deformation) is most easily understood by introducing a macroscopic damage parameter D defined by

$$D = \frac{1}{\overline{E}_A} - \frac{1}{\overline{E}_A^o} \quad . \tag{1}$$

The parameter $\overline{E}_A$ is the effective axial modulus of a laminate (constrained so that in-plane shear strain is zero) having ply cracks, while $\overline{E}_A^o$ ($> \overline{E}_A$) is the corresponding value for the same laminate without ply cracks. Clearly D is zero when the laminate is undamaged. It has been shown [1-3] that the following relations define, in terms of D, the values of the various thermoelastic constants of a damaged laminate which are satisfied for ANY state of cracking in the plies of a general symmetric laminate (see [2, 3] for formal definitions of the various effective thermoelastic constants)

$$\frac{1}{\overline{E}_t} - \frac{1}{\overline{E}_t^o} = (k')^2 D \,, \quad \frac{1}{\overline{E}_A} - \frac{1}{\overline{E}_A^o} = D \,, \quad \frac{1}{\overline{E}_T} - \frac{1}{\overline{E}_T^o} = k^2 D \,, \tag{2}$$

$$\frac{\overline{\nu}_t^o}{\overline{E}_T^o} - \frac{\overline{\nu}_t}{\overline{E}_T} = k\,k' D \,, \quad \frac{\overline{\nu}_A^o}{\overline{E}_A^o} - \frac{\overline{\nu}_A}{\overline{E}_A} = k D \,, \quad \frac{\overline{\nu}_a^o}{\overline{E}_A^o} - \frac{\overline{\nu}_a}{\overline{E}_A} = k' D \,, \tag{3}$$

$$\overline{\alpha}_t - \overline{\alpha}_t^o = k' k_1 D \,, \quad \overline{\alpha}_A - \overline{\alpha}_A^o = k_1 D \,, \quad \overline{\alpha}_T - \overline{\alpha}_T^o = k\,k_1 D \,. \tag{4}$$

These relations, derived by considering ply crack closure for uniaxial loading in the axial, in-plane transverse and through-thickness directions, define effective anisotropic properties such as Young's moduli (E), Poisson's ratios (ν), and coefficients of thermal expansion (α). The properties E and α are different in three orthogonal directions denoted by t – through-thickness direction, A – axial direction, and T – the in-plane transverse direction. The suffices t, A and T are also used to define anisotropic Poisson's ratios, the precise definition of which may be determined from stress-strain relations. The three laminate constants k, k' and k_1 may be calculated [2, 3] from the properties the plies, and the reduced thermoelastic properties of the laminate in an undamaged state. It should be noted that k and k' do not depend on thermal expansion coefficients. A comparison of model predictions for the axial Young's modulus of various types of cross-ply laminate as a function of ply crack density, with experimental data [5] for both carbon fibre reinforced plastic (CFRP) and glass fibre reinforced plastic (GRP) is given in see Ref. [3]. The excellent agreement observed for values of axial modulus is expected also for other thermoelastic constants, because of the validity of the inter-relationships (1-4). Homogenisation techniques [6] are used to replace the effects of discretely cracked plies by homogenised layers having reduced properties. This technique is used to represent ply cracking in more than one orientation for the case of general symmetric laminates.

Energy based ply crack formation

While ply failure is often predicted using a strength argument (as can be seen from the design criterion known as first ply failure), this approach cannot be justified using physical principles. It is merely a convenient engineering approach that has been used for many years. Our approach is to examine the application of a key physical principle, energy balance, to the ply cracking problem,

and to develop relatively simple formulae that can be used to replace the conventional stress-based first ply failure approach. For isothermal conditions, the principle of energy balance equates the increment of work δW by the externally applied tractions to the various types of energy that can be associated with the system, as follows,

$$\delta\Gamma + \delta K + \delta U + \delta D = \delta W \, . \tag{5}$$

The energy contributions are: δU the increment of stored elastic energy (strictly for isothermal conditions, this is a thermodynamic function known as the Helmholtz free energy), δK the increment of kinetic energy which is never negative, δD an increment of dissipated energy (a positive quantity) if some irreversible processes are occurring such as plastic flow or creep deformation (both relevant mechanisms for polymer composites), and finally $\delta\Gamma$ the energy absorbed in the regions of the ply crack surfaces as the crack forms (related to the fracture energy for ply cracking). Assuming that ply cracks form under fixed applied tractions (rather than fixed applied displacements), and on making use of the fact that kinetic energy is never negative, the general energy balance equation may be recast in a simple inequality involving the increment of Gibbs free energy δG, namely

$$\delta\Gamma + \delta G < 0 \, . \tag{6}$$

In practice non-uniform ply cracking in laminates is usually observed where the ply cracks form one at a time. The simulation of such ply cracking is achieved by assuming that in the plies of the laminate there are a set of potential ply crack formation sites where ply cracks may form during loading. In any simulation only a fraction of the sites will actually be cracked. Such behaviour has been modelled by allowing the fracture energy for ply cracking to be statistically distributed. This is achieved by assuming that the fracture energy at each potential ply crack formation site is taken at random from a normal distribution that is characterised by its mean value and standard deviation. Energy methods are used to determine the ply cracking sequence. Assuming that n ply cracks have already formed in the laminate, an energy calculation is performed at each uncracked site to determine the stress needed to form the $(n+1)^{th}$ crack at that location. Having 'tested' all remaining uncracked sites, the site requiring the least stress to form the crack is taken to be the site at which the $(n+1)^{th}$ crack forms. This procedure is repeated successively as damage progressively increases during loading. The theory to account for non-uniform ply cracking has been described in Ref. [7] where an approximate method is used in the model to: i) develop a method of calculating the thermoelastic constants of non-uniformly cracked laminates, and ii) generate simple analytical results for first ply failure stress. The approximation has been compared with accurate numerical solutions (see ref. [7]) and found to be excellent for a wide range of conditions.

Consider progressive cracking between two distinct damage states 1 and 2 having ply crack patterns defined by:

$$\text{State 1} \;=\; \{L_1,\, L_2, \ldots L_n\} \, , \qquad \text{State 2} \;=\; \{L_1,\, L_2, \ldots L_{n+1}\} \, ,$$

and leading to energy absorptions Γ_1 and Γ_2 respectively, involving the fracture energy γ for ply cracking, where sequences L_i denote the separations between neighbouring ply cracks. On making use of the simple expression for the Gibbs free energy per unit volume in the energy balance inequality, the effective stress s, needed to enable the steady state propagation of a ply crack, can be calculated for triaxial stress states using the following simple formula (valid in the absence of shear deformation) [1-3]

$$s > \sqrt{\frac{2\left[\Gamma_2 - \Gamma_1\right]}{\dfrac{1}{\overline{E}_A^{(2)}} - \dfrac{1}{\overline{E}_A^{(1)}}}} + s_c \, , \quad n \geq 0 \, , \quad \text{where } \; s = k'\sigma_t + \sigma + k\sigma_T \, , \tag{7}$$

where s_c is the value of s when the ply cracks just close in the ply where they are being modelled discretely.

The modelling described for the prediction of ply cracking in a general symmetric laminate has been implemented in a software system known as PREDICT that is shortly to become a new module for the NPL CoDA software system (Composites Design and Analysis - see the web page http://www.npl.co.uk/npl/cmmt/cog/coda.html).

Examples of applications to layered systems

The PREDICT software system has been used to predict, for both GRP (Silenka/epoxy MY750) and CFRP (AS4/epoxy 35) cross-ply laminates, the effect on first ply failure stress of the thickness of the individual plies. The laminates, all of total thickness 4 mm, that have been considered are $[0_8/90_8]_s$, $[0_4/90_4]_{2s}$, $[0_2/90_2]_{4s}$ and $[0/90]_{8s}$ where both GRP and CFRP systems are assumed to have a ply thickness of 0.125 mm and a fracture energy for ply cracking of 240 J/m^2. Thermal residual stresses are included in the model based on a value $T - T_o = -85^oC$, where T is the temperature of simulation and where T_o is the stress-free temperature for the laminate. Ply refinement techniques were used to increase the accuracy of the through-thickness stress and displacement distributions by sub-dividing each ply into 5 sub-elements of equal thickness.

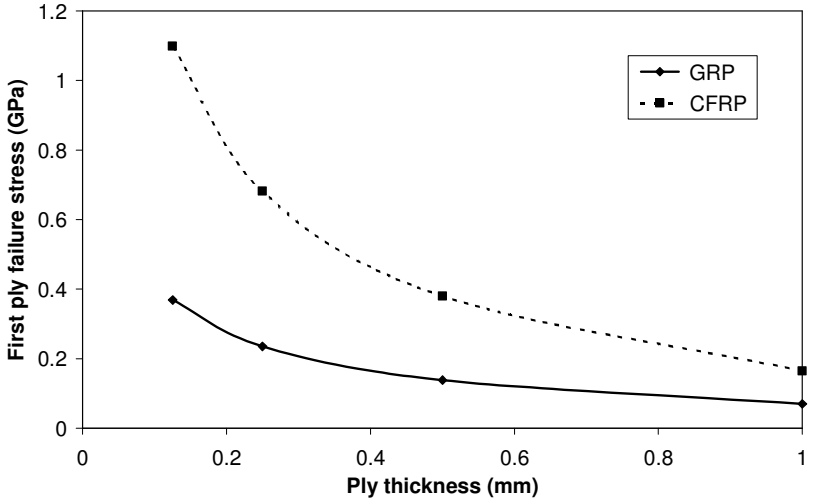

Fig.1 : Effect of ply thickness on first ply failure for GRP and CFRP laminates

The results for first ply failure stress are shown in Fig.1 where it is seen that decreasing the ply thickness while maintaining the total thickness of the laminate at 4 mm leads to dramatic increases in the resistance of the laminates to ply failure. The CFRP laminates perform better, although fibre failure might determine the occurrence of initial damage when the plies are very thin.

Another example of an application of the energy-based methodology for failure prediction concerns the first cracking of interleaved layers of titanium nitride and steel forming a $[TiN/Steel]_{4s}$ laminate. The layer thickness, laminate thickness and simulation length are all reduced by factors of ten from a thickness of 1 mm down to a thickness of 100 nm. The simulations assume a value $T - T_o = -500^oC$. Layer refinement techniques were used to increase the accuracy of the through-thickness stress and displacement distributions by sub-dividing each layer into 5 sub-elements of equal thickness. The results shown in Fig.2 exhibit a dramatic increase in the first cracking stress as the layer thickness is reduced, illustrating a well-known nano-effect where decreasing length scales lead to improved performance. It must be emphasised that, although strain relaxation cannot be

achieved by cracking, other mechanisms such as dislocation formation, will occur invalidating the predictions of Fig.2 when the layer thickness is in the micrometer range. Also, cohesive zones near crack tips, modelling inter-atomic interactions across the fracture surface, may need to be modelled as the layers become very thin.

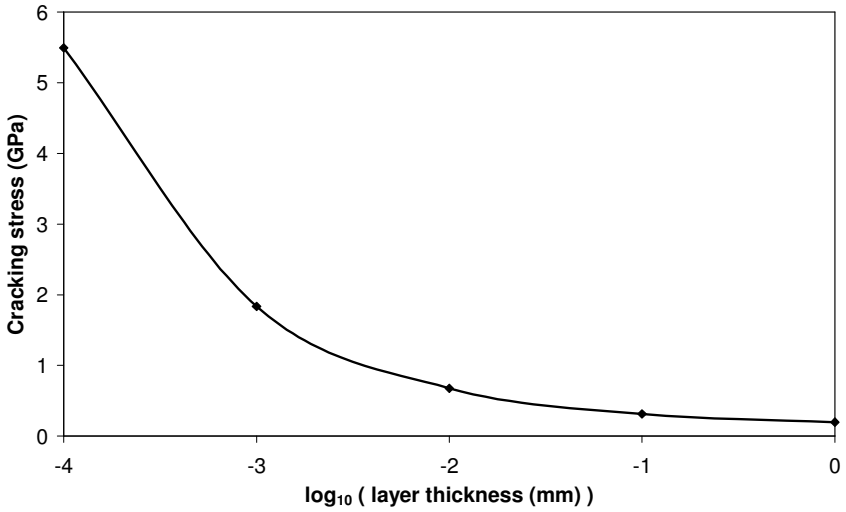

Fig.2 : Size effect on cracking stress for a titanium nitride/steel layered system.

The development of reliable models for laminate strength prediction is in its infancy, although good progress is being made, as demonstrated here by the ability of physically based models to predict laminate thickness effects. An important feature of both Figs.1 and 2 is that an energy-based methodology for cracking in layered materials is leading to strong thickness dependencies observed in practice, and such dependencies cannot be predicted using a strength of materials approach that is the basis of widely used conventional failure criteria. This issue should be a concern for engineers involved with the design of both laminated structures and other layered systems. It is again emphasised that the effects of ply thickness, and laminate lay-up, were not considered in the International Failure Prediction Exercise (Ref. [4]), and that the energy-based model described here is the only one [1] of those included in the Exercise that is able to take such effects into account.

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