Composite Material Model with Circular Inclusion under Imperfect Interface Conditions

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Summary

A constitutive model for a composite material of dispersed particles is developed, on which imperfections of the particle interface is modeled by employing a spring layer with both sliding and the debonding deformations. The imperfection is modeled with the use of Gao's stress function solutions for the spring layered between the particle and the matrix. Especially, a present double equivalent inlucion modeling enabled the use of the Gao's solution to the Eshelby's equivalent method. The present model is applicable for both the particle interface degrading problem and the surface reinforcing problem of the composite.

Introduction

Efforts have been made, in general, to develop the macro average material constitutive law for the particle dispersed composite material. Hashin-Schtrikman's theory[1] based on the variational principle has been known to give upper and lower bounds for multiphase composite of isotropic elasticity. Since the equivalent inclusion theory for one ellipsoidal particle in an isotropic infinite domain was established by Eshelby [2], a number of applications have been made in the area of the composite material, the micro mechanics fields, etc., with the form of Eshelby tensor. The mean field theory by Mori-Tanaka [3] that extends the Eshelby's equivalent theory to the particle dispersed composite material is in a closed form, is thus often used as an averaging method of the composite material. When the volume fraction of particle is not small, or when constituents' material rigidities differ notably, stress interaction between particles is not negligible , thus it is pointed out by Mura [4] that the accuracy of the model falls. The mean-field theory has even been utilized for modeling the composite where the matrix undergoes elasto-plastic deformation, by Tandon-Weng [5].

Self-consistent (SC) scheme, on the other hand, provides a concept of treating the media surrounding a particle as a homogeneous material with the averaged characteristics of the composite to be determined, and has the advantage that the above-mentioned problem can be avoided. On this SC concept, an elastic constitutive model with spherical inclusions was developed [6] followed by elasto-plastic model [7]. The validity of those models developed was verified by excellent agreements between models and the analysis of a meso-scale particle scattered domain using FEM.

The above is the case where the interface between the inclusion and the matrix is in perfect adhesion. When an imperfect condition exists in the interface, and moreover when its effect on the macroscopic characteristics is considered to be non-negligible, the models mentioned in the above cannot be applied as they are. Although the interfacial exfoliation between the inclusion and the matrix in a fiber reinforced composite material or the crystal slide of grain in metals is thought in general to bring an interfacial imperfection, those phenomena may not necessarily be detrimental; the fracture toughness of a composite may sometime be increased, or inter-grain sliding may become the factor of the super-plasticity phenomenon in an alloy, and can be advantageous.

For modeling the interfacial imperfection, a spring element layer of thickness zero has been

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considered and addressed by other researchers. For those problems also, the equivalent inclusion method can serve as an effective scheme to determine the macro behavior of the comoposite.

For the use of the equivalent inclusion modeling for the imperfection-spring problem, Qu [8] derived an Eshelby tensor using the Green's function. However, the applicable range is limited to the case when the compliance of the spring is sufficiently low compared with that of the matrix material. Although this method is effective in problems such as bonded interface, it is not effective for the problem of weakening interfacial conditions, as seen in exfoliation.

Gao [9], considering even interfacial exfoliation and slide, obtained the particle solution using stress functions effective for all the spring compliance cases. He also gave a solution for the imperfect particle subjected to an eigen strain. Although it is for a two-dimension circular particle, Gao's solution is the applicable only solution of the spring, so that this paper aims at building an equivalent inclusion model using his solution.

In the present effort, consider a macroscopically homogeneous composite material with randomly dispersed meso-scopic particles. The objective is to develop a macro-constitutive model for the composite when particles undergo interfacial weakening, i.e., exfoliation and slide, as well as interfacial bonding.

Theoretical Model

In the authors' previous work, the two-fold equivalence modeling was performed. Consider a meso-domain in a composite, whose averaged macro-compliance, yet to be determined, is denoted by $\overline{\Gamma}$. The quantity with $(\,\overline{\ }\,)$ denotes that of the averaged value to be determined according to the self-consistent concept. An inclusion particle is embedded in the averaged medium as shown in Figure 1(A), where a spring layer is modeled between the inclusion and the surroundings. The model in (A) can be made equivalent to the model (B) in Figure 1, where the spring layer keeps unchanged but the properties of the core are replaced by those of the surrounding medium and an eigen strain, ε^{i*}, imposed to it. Further, the equivalenced inclusion and the spring layer together are again equivalenced to

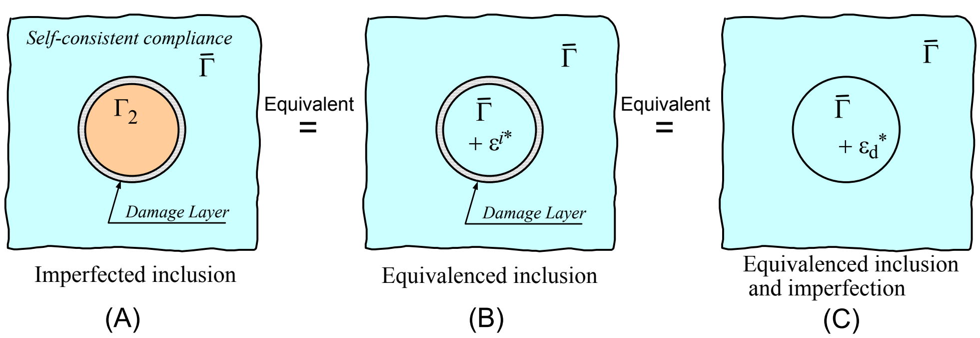

Fig. 1 Composite with imperfect interface and its equivalent domain

another virtual inclusion with an eigen strain, ε_d^*, as shown in (C). In these process, the macro strain-stress relationship of elastic composite with the modeled interfacial spring layer was derived as the final form of the composite constitutive law such that,

$$\varepsilon_0 = \overline{\Gamma} : \sigma_0 \tag{1}$$

$$\overline{\Gamma} = \Gamma_1 - f(\Gamma_1 - \Gamma_2) : \boldsymbol{B}_0 + f(\boldsymbol{D}^{\sigma} + \boldsymbol{D}^* : \boldsymbol{A}_0) \tag{2}$$

The **BOLD** face letters denote vectors or tensors, while (:) means the double inner product. Those with subscripts 1 and 2 correspond the matrix and the inclusion materials, respectively. f is the volume fraction of the inclusions. Quantities in the above are defined such that,

$$\boldsymbol{A_0} = [(\boldsymbol{\Gamma_2} - \overline{\boldsymbol{\Gamma}})^{-1} - \overline{\boldsymbol{E}}_{\boldsymbol{0}} : (\boldsymbol{I} \otimes \boldsymbol{I} + \boldsymbol{D}^*)]^{-1} : (\boldsymbol{I} \otimes \boldsymbol{I} + \overline{\boldsymbol{E}}_{\boldsymbol{0}} : \boldsymbol{D}^{\sigma}) \tag{3}$$

$$\boldsymbol{B_0} = \boldsymbol{I} \otimes \boldsymbol{I} + \overline{\boldsymbol{E}}_{\boldsymbol{0}} : [(\boldsymbol{I} \otimes \boldsymbol{I} + \boldsymbol{D}^*) : \boldsymbol{A_0} + \boldsymbol{D}^{\sigma}] \tag{4}$$

$$\overline{\boldsymbol{E}}_{\boldsymbol{0}} = \overline{\boldsymbol{E}} : (\overline{\boldsymbol{S}} - \boldsymbol{I} \otimes \boldsymbol{I}) \tag{5}$$

where, $\overline{\boldsymbol{S}}$ stands for the Eshelby tensor for the average composite.

The damage tensor $\boldsymbol{D}^{\sigma}$ is due to the external loading; the nonzero components of the tensor will be described such that,

$$D^{\sigma}_{1111} = D^{\sigma}_{2222} = \frac{\kappa_1 + 1}{2\mu_1}\left[\frac{1}{t\kappa_2 n - tn + 2n + 4} + \frac{(k+n)(1+t\kappa_2)+12}{P}\right] \tag{6}$$

$$D^{\sigma}_{1122} = D^{\sigma}_{2211} = \frac{\kappa_1 + 1}{2\mu_1}\left[\frac{1}{t\kappa_2 n - tn + 2n + 4} - \frac{(k+n)(1+t\kappa_2)+12}{P}\right] \tag{7}$$

$$D^{\sigma}_{1212} = \frac{\kappa_1 + 1}{2\mu_1}\frac{(k+n)(1+t\kappa_2)+12}{P} \tag{8}$$

While, the other tensor $\boldsymbol{D}^*$ represents the damage due to the assignment of the eigen strain ε^{i*} to the equivalent core inclusion, and will be given by,

$$D^{*}_{1111} = D^{*}_{2222} = -\frac{2}{t\kappa_2 n - tn + 2n + 4} - \frac{(k+n)(1+t\kappa_2)+12}{P} \tag{9}$$

$$D^{*}_{1122} = D^{*}_{2211} = -\frac{2}{t\kappa_2 n - tn + 2n + 4} + \frac{(k+n)(1+t\kappa_2)+12}{P} \tag{10}$$

$$D^{*}_{1212} = -\frac{(k+n)(1+t\kappa_2)+12}{P} \tag{11}$$

In the above, the material parameters are defined in the following.

$$t = \frac{\mu_1}{\mu_2} \tag{12}$$

$$\kappa_1 = \begin{cases} 3 - 4\nu_1 \\ \dfrac{3-\nu_1}{1+\nu_1} \end{cases} \qquad \kappa_2 = \begin{cases} 3 - 4\nu_2 & \text{: for plane stress} \\ \dfrac{3-\nu_2}{1+\nu_2} & \text{: for plane strain} \end{cases} \tag{13}$$

$$P = 3p_1 p_2 + knp_2 p_3 + p_3 p_1 + 12 \tag{14}$$

$$p_1 = k + n, \quad p_2 = t + \kappa_1, \quad p_3 = t\kappa_2 + 1 \tag{15}$$

where, n and k are normal and tangential spring coefficients, respectively.

The normal and the tangential stresses on the interface are defined in relation to the displacement gaps between the matrix and the inclusions across the interface that,

$$\sigma_{rr}(a,\theta)=\frac{n\mu_1}{a}\Delta u_r(a,\theta) \qquad (16)$$

$$\sigma_{r\theta}(a,\theta)=\frac{k\mu_1}{a}\Delta u_\theta(a,\theta) \qquad (17)$$

where a denotes a particle diameter. The condition, $k=0$ and $n=\infty$, represents the perfect sliding with no delamination, and $k=\infty$ and $n=\infty$ corresponds to the perfectly bonded interface.

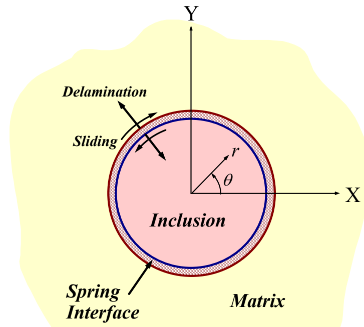

Fig. 2 Inclusion particle geometry with spring layer

Results

Present model with the inclusion interface imperfection is numerically checked for its behavior and reliability to serve as a constitutive law. For this purpose, a finite element analysis of a representative two-dimensional composite domain was performed with the same material condition. The self-consistent condition assumed in the above model was accounted for by assigning a periodic boundary conditions in the finite element analysis. The following case of matrix and incblusion material properties is considered :

Young's modulus : $E_1=2.0GPa$ (Matrix) $E_2=40.0GPa$ (Inclusion)

Poisson's ratio : $\nu_1=0.35$ (Matrix) $\nu_2=0.18$ (Inclusion)

Figure 3 shows the calculated results(denoted by SCC) from the present model compared with those by the finite element results, for various cases of volume fraction of the inclusion with its interfacial conditions. In the present FEM, the normal and the tangential springs are also modeled around the each inclusion particle. Several cases of inclusion volume fraction were computed for each conditions.

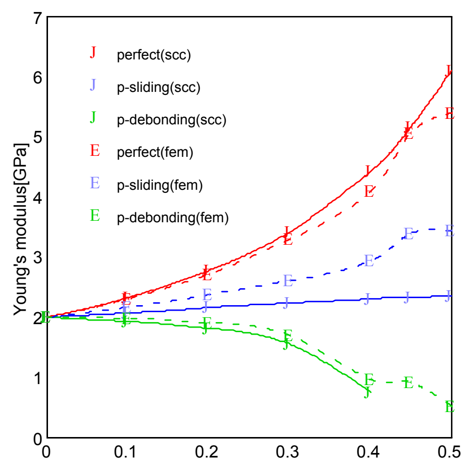

Fig. 3 Perfect debonding and perfect sliding compared with FEM

For the perfect interface condition, in general, a good agreement of the present model compared with the FEM results is seen to be preserved. Lesser accurate FEM-results observed in the higher volume fraction region maybe due to the interaction among inclusions, which probably are not fine enough. Since the rigidity of the inclusion is 20 times greater than that of the matrix, the composite rigidity rises rapidly for increasing volume fraction.

Results of the perfect debonding with no sliding are shown in green symbols providing reasonable agreements with the corresponding FEM results. Very significant drop of Young's modulus from the perfect inclusion case is recognized. In both the results, the global Young's modulus decrease as the volume fraction increase, showing the fact that the effect of the debonding on the interface is far greater than that of the reinforcement due to adding the stiffer inclusions.

Results of the perfect sliding with no debonding does not seem exhibiting much variation for the volume fraction change, however the effect of sliding itself on the rigidity is noteworthy as recognizing

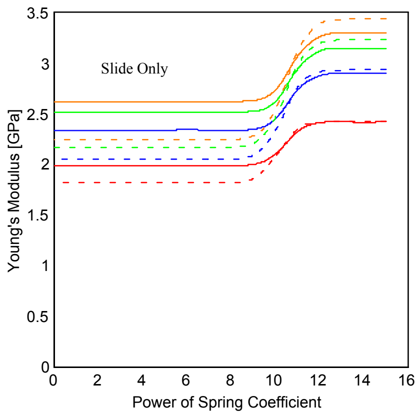

Fig. 4 Global Young's modulus due to sliding

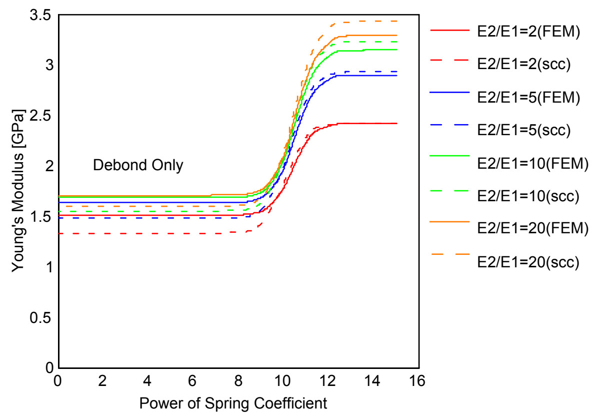

Fig. 5 Global Young's modulus due to debonding

the fall of Young's modulus from the perfectly bonded results. Reason for the discrepancies between the present sliding results and those of the FEM could be in the modeling of the spring in the FEM; spring elements are located at each node unlike in the present SCC model, where the continuous spring layer is located.

Tested in the numerical analysis was the effect of the spring stiffness on the global composite material characteristics. Figures 4 and 5 show the Young's modulus of the composite for varied spring power coefficient, respectively for the cases sliding only and debonding only while the other spring coefficients are kept significantly stiffer. In both the figures, Young's modulus varies only in the range where the spring stiffness is comparable with those of the matrix or the inclusion materials; in other ranges it shows no variation, i.e. almost the perfect bonding is secured when the power coefficient is greater than 12, while the perfect sliding or debonding is performed when smaller than 8 or 9 for all the rigidity ratio, E_2/E_1.

In sliding case presented in Figure 4, as it was seen in the previous figure also, correlation between the present model and the FEM results is not that excellent in quantity, but their behaviors are alike, and the drop of the stiffness is significant. Thus, the effect of the particle surface sliding is proven to be great even though there is no accompanied delamination on the surface. This sliding effect of the global composite behavior would suggest a possible explanation to a super plastic material characteristics in highly ductile metals, for which crystal grain slipping at the boundary is considered to be responsible.

Shown in Figure 6 is the results of the SCC and the FEM on the global Poisson's ratio. When the spring is sufficiently stiff, for both the 'Slide only' and the 'debond only', the global Poisson's ratio converges to the value around 3.0, which is the mixed average of the matrix and the inclusion materials. Again, in the range where the spring stiffness is comparable with those of the matrix or the inclusion material, the global Poisson's ratio varies meaningfully. It is noted that the Poisson's ratio for the 'slide only' rises as the sliding spring becomes softer, providing rather like plastic nature even with no plastic deformations assumed nor occurring in either the matrix or the inclusions.

For the 'debond only' case, Poisson's ratio drops radically when the radial surface spring gets softer; the values of Poisson's ratio become even below zero when it is sufficiently soft. This rather peculiar results seem unreasonable, being suspected involving some errors in the model. But totally independent FEM results also provide similar results with negative Poisson's ratio as well. This is because the problem setting of the 'debond only' prohibits sliding, allowing only radial expansion, which would be unnatural but possible, though, to generate a special material with the negative Poisson's ratio.

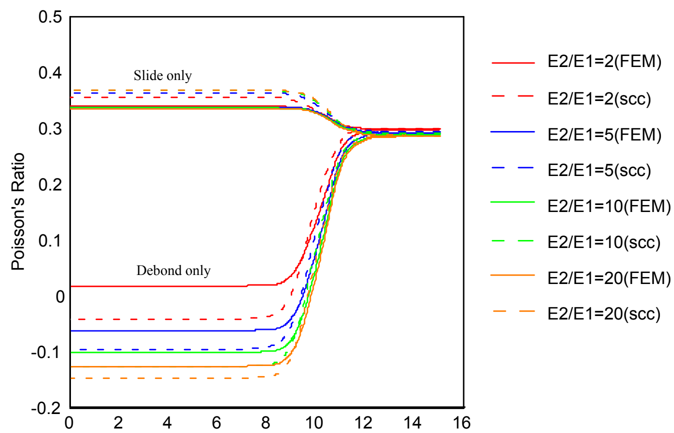

Fig. 6 Global Poisson's Ratio due to sliding and debonding

Conclusions

A constitutive model for a composite of dispersed particles is developed, on which imperfections of the particle interface is modeled by employing a spring layer with both sliding and the debonding deformations. The imperfection is modeled with the use of Gao's stress function solutions for the spring layer between the particle and the matrix. Especially, a present double equivalent inlucion modeling enabled the use of the Gao's solution to the Eshelby's equivalent method. The present model is applicable for both the particle interface degrading problem and the surface reinforcing problem of the composite. As to sliding, the present model can be extended to model a micro mechanism for a super-plasticity.

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