High Frequency Scattering of Steel Bars with Embedded Cavities via BEM

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Summary

This paper uses a boundary element method formulation (BEM) to compute high frequency wave propagation in a solid layer enclosing a small cavity. This model includes analytical solutions derived by the authors for computing the three-dimensional displacements in a flat solid elastic stratum with free top surfaces, when it is subjected to a spatially sinusoidal harmonic line load. This solution avoids the discretization of the horizontal surfaces of the layer and only the boundary of the cavity needs to be discretized by boundary elements. The formulation is performed in the frequency domain for varying spatial wave numbers in the axial direction of the layer for which the geometry does not vary. Time results are obtained by means of inverse Fourier transforms, to help understand the variation of the displacement field.

Introduction

The existence of internal defects as voids or cracks in a structure, can affect its integrity and its dynamic behavior. The detection of possible defects in engineering materials or structural elements has become a subject of interest for investigators along the years. Experimental and theoretical studies have been conducted in order to understand the performance of damaged materials or structures. Among the non-destructive techniques, ultrasound, based on the interaction between elastic waves and defects, is widely used for structure inspection. The measured data are analyzed and interpreted in order to detect the location and the size of damaged zones.

However, to deal with the indirect problem it is necessary to thoroughly understand all the phenomena involved. Thus, modeling the direct or the inverse problems is a way to develop methods of inspection and data analysis. Different methods have been applied over the years to model this problem. Recently published works include the use of hybrid finite element-boundary element method [1], the effective medium method [2], the Born and the extended quasistatic approximations in parallel, with measurements [3], a compact matrix methodology based on finite element discretization in wavenumber space and fast Fourier transform [4] and mixed methods (volume integral equation with boundary integral equation method) [5].

In this paper, high frequency excitation is used to study wave propagation in an elastic layer containing small cavities. The numerical simulation is performed using a

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direct BEM formulation which utilizes 2.5D Green's functions that were developed in a previous work [6] to simulate the wavefield produced in a homogeneous three-dimensional free solid layer formation of infinite extent, subjected to a spatially sinusoidal harmonic line load, polarized along the horizontal, vertical and z directions. As the 2.5D Green's functions simulate the free horizontal surfaces of the layer, only the boundaries of the cavity need to be discretized by boundary elements.

The 2.5D formulation refers to a problem where the source is 3D, but the geometry does not vary along one direction (2D). Thus a three dimensional problem can be formulated as a summation of two-dimensional problems for varying wavenumbers along the z direction, following the technique of Bouchon [7] and Kim et al. [8]. The Green's functions for each two-dimensional problem were computed as a continuous superposition of plane waves with different wave numbers in the x direction (k_n), adopting the approach used first by Lamb [9].

The formulation is established in the frequency domain for different spatial wavenumbers in the z dimension. The time domain responses are obtained by applying an inverse Fourier transform to the frequency results. A small imaginary part is introduced into the frequency to avoid aliasing phenomena [10].

Boundary Element Formulation

The Boundary Element Method (BEM) is formulated in the frequency domain (ω) to compute the 3D wavefield produced by a spatially sinusoidal harmonic line load within a solid elastic layer containing a small cavity.

The BEM algorithm, developed for this work, includes analytical solutions, derived by the authors, to simulate wave propagation in a free layer solid formation produced by a spatially sinusoidal harmonic line (steady state) load [6]. In this context, only the free boundary of the cavity, where null traction boundary conditions are required, needs to be discretized by boundary elements. Thus, the boundary integral equation when an incident wave excites the system is

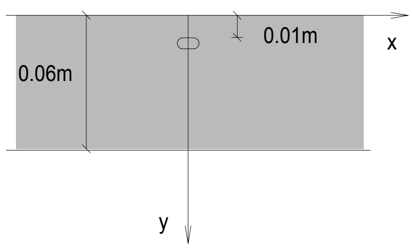
$$-\int_C u_j(\boldsymbol{x},\nu,\omega)\, H_{ij}(\boldsymbol{x},\boldsymbol{x}_0,\omega)\, ds + u_i^{inc}(\boldsymbol{x}_0,\omega) = c_{ij} u_i(\boldsymbol{x}_0,\omega) \quad (1)$$

where, $i, j = 1, 2$ are the normal and tangential directions in relation to the boundary surface; $i, j = 3$ indicates the z direction; $H_{ij}(\boldsymbol{x},\boldsymbol{x_0},\omega)$, are the tractions in direction j at x, on boundary C, originated by a unit sinusoidal line load acting at the source, $\boldsymbol{x_0}$, in direction i; vector ν is the unit outward normal at the boundary, $u_j(\boldsymbol{x},\nu,\omega)$ are the displacements to be determined in the boundary and $u_i^{inc}(\boldsymbol{x}_0,\omega)$ are the displacements generated by the incident wave.

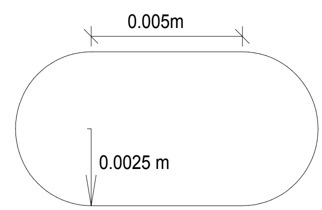
In this formulation the incident field includes the influence of the free layer solid formation. Thus, in equation (1) the incident displacements are computed analytically as the sum of the source terms (calculated for a two-and-a-half dimensional full-space) and the surface terms, originated at both free horizontal surfaces of the layer (corresponding to the reflections from those surfaces). The integral equations are combined and subjected to the boundary conditions, and discretized appropriately. A system of equations is established and solved for the nodal displacements. The present BEM algorithm was implemented and verified by comparing the results with those provided by a BEM code, which requires the full discretization of all boundaries of the system.

Numerical Application

The BEM formulation is used to simulate wave propagation inside a solid layer including a small cavity, generated by a plane source located close to the bottom surface of the layer and acting in the normal direction of the layer. The solid layer is 0.06m thick (see Figure 1a) while the top of the cavity is 0.01m below the upper layer's surface, and it has the geometry and size illustrated in Figure 1b.



a)



a)

Figure 1. Geometry of the model: a) solid layer with cavity; b) geometry of the cavity.

The solid properties allow a shear wave velocity of $\beta = 3191\,\text{m/s}$ and a dilatational wave velocity of $\alpha = 5970\,\text{m/s}$ having a density of $\rho = 7850\,\text{Kg/m}^3$. The boundary of the cavity is discretized using a number of boundary elements defined according to the excitation frequency of the harmonic source. The ratio between the wavelength of the incident waves and the length of the boundary elements is considered equal to 10. The number of boundary elements used to model the inclusion was never less than 70.

The system is excited by a plane source acting $0.2\,\text{mm}$ above the bottom surface of the layer ($y = 0.0598\,\text{m}$). The solid displacements are computed over a grid of 61 by 40 receivers along the x and y directions respectively. The receivers are 0.015 m apart in the two directions and are placed from $x = -0.045\,\text{m}$ and $y = 0.00075\,\text{m}$ to $x = 0.045\,\text{m}$ and $y = 0.05925\,\text{m}$ .A total of 1024 frequencies were computed in the range of [1000 Hz, 1024000 Hz], with an increment of 1000 Hz, allowing a total observation time of $T = 1\,\text{ms}$. Complex frequencies of the form $\omega_c = \omega - i\eta$ (with $\eta = 0.7\Delta\omega$) are used to prevent aliasing phenomena. The responses in the frequency domain are transformed to the time domain by applying an inverse Fourier transform. The source is modeled as a Ricker pulse with a characteristic frequency of 290 kHz.

The amplitude of the displacements registered at the grid of receivers is plotted in a grayscale ranging from black to white, as the amplitude increases. To illustrate the system behaviour for the first reflections, a sequence of snapshots is presented (see Figure 2).

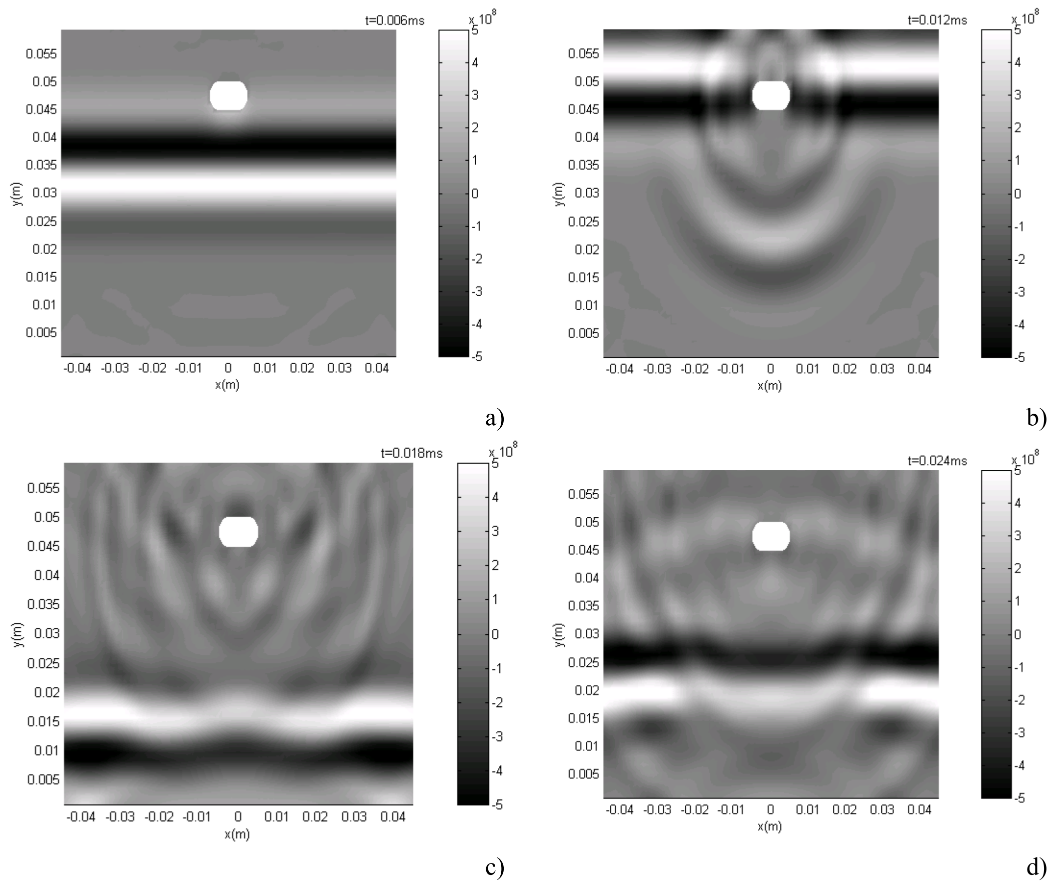
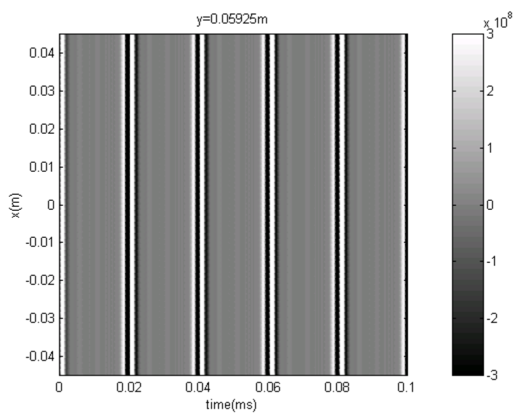


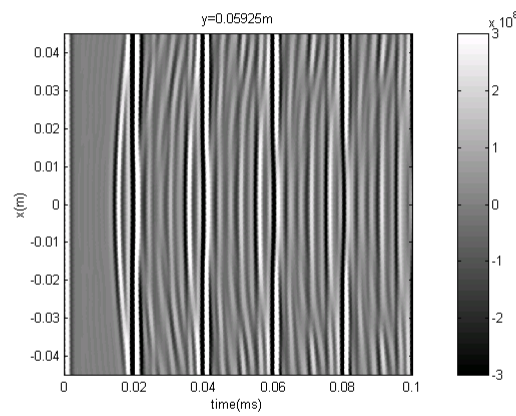
Figure 2. Time displacements over a grid of receivers for a characteristic frequency of 290 kHz: a) t=0.006ms; b) t=0.012ms; c) t=0.018 ms; d) t=0.024 ms.

In the first plot (Figure 2a), at t=0.06 ms, the incident wave has reached the bottom of the cavity and a small perturbation is visible due to the wave field starting to be scattered by the cavity. Figure 2b shows the waves descending after being reflected from the top surface of the solid layer and being reflected again by the cavity. As the time passes the waves propagate away from the cavity, and the influence of the reflections on its boundary is clearly visible in Figure 2c. The waves hit the bottom surface of the solid layer and are reflected back towards the top surface, as can be seen in Figure 2d. The process continues and the wave field becomes more and more complex as result of the multi reflections at the boundaries of the cavity and the solid layer.

The time displacement amplitudes registered along the lower line of receivers are given to show the time evolution of the wave field. Figure 3a shows the time displacements when the solid layer has no cavity and Figure b shows the results when the cavity is present. The interferences caused by the cavity in the displacement wave field can be seen quite clearly in Figure 3b.



a)



b)

Figure 3. Time displacements over a line of receivers: a) elastic solid layer; b) elastic solid layer with a cavity.

Conclusions

The BEM algorithm, incorporating the Green's functions for a free solid layer, was found to be an efficient method for simulating high frequency propagation in a solid layer which contained a small cavity. The perturbation provoked by the cavity in the wave field is plainly visible in all the computations performed.

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