PREDICTION METHODOLOGIES FOR NON-STATIONARY TURBULENT FLOWS

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Abstract

Ever-increasing demands on the performance of Reynolds-averaged Navier-Stokes (RANS) models for industrially relevant flows has been the impetus for development of closure models that replicate more details of the physical processes that occur in such turbulent flows. Alternative formulations such as large eddy simulation (LES) often provide such additional details but at increased computational cost. Hybrid formulations have now been developed to ease the computational burden of LES. Some current methodologies are discussed within a unified framework and some results from benchmark cases are presented.

It is currently necessary for numerical calculation of practical engineering turbulent flow fields to solve a set of equations for flow variables that represent the motion of a limited spectral range of scales. This description holds true for LES, RANS formulations and any of the newly developed hybrid or composite methodologies currently being proposed. As such, the equations describing the filtered motions in any of these formulations are form-invariant. They obviously differ with respect to the flow field motions being described, and this is predicated on how the higher-order correlations are parameterized.

As is customary in all these formulations, the flow variable f is decomposed into a filtered part, $\overline{f}$, and a sub-filtered part, f', given as $f = \overline{f} + f'$. Generally, the filtering process can be defined as a subset of the general operation (e.g. Sagaut 2006)

$$\overline{f}(\mathbf{x},t) = G * f = \int G(\mathbf{x}-\mathbf{x}', t-t')\, f(\mathbf{x}',t')\, \mathrm{d}\mathbf{x}'\, \mathrm{d}t' \, . \tag{1}$$

Different forms for the convolution kernel can be associated with the various solution methodologies. For the Reynolds-averaged formulation, for example, stationarity is usually assumed and a long time average corresponds to an ensemble average with a filter function given by

$$G_{\mathcal{T}}(\mathbf{x},t) = G(\mathbf{x})\, G_{\mathcal{T}}(t) = \delta(\mathbf{x})\, \frac{1}{\mathcal{T}} \mathcal{H}(\mathcal{T}-t)\, \mathcal{H}(t) \, , \tag{2}$$

such that $\overline{f}_{\mathcal{T}}(\mathbf{x},t) = G_{\mathcal{T}} * f = \mathcal{T}^{-1} \int_{t-\mathcal{T}}^{t} f(\mathbf{x},t')\mathrm{d}t'$. In such flows, the entire spectral range of scales is modeled so the sub-filtered part f' is a fluctuating quantity whose average is zero $\overline{f'} = 0$, and the mean quantity $\overline{f}$ can be extracted from

$$E\{f(\mathbf{x},t)\} = \lim_{\mathcal{T}\to\infty} \overline{f}_{\mathcal{T}}(\mathbf{x},\mathcal{T}) = \lim_{\mathcal{T}\to\infty} \frac{1}{\mathcal{T}} \int_0^{\mathcal{T}} f(\mathbf{x},t)\, dt, \tag{3}$$

Thus, Reynolds averaging can be considered as the convolution filter $G_{\mathcal{T}}$ with a sufficiently large $\mathcal{T}$. For a flow statistically periodic in time (cyclo-stationary), it is customary to use some form of phase averaging, corresponding to a filter function given by

$$G_T(\mathbf{x},t) = G(\mathbf{x})\; G_T(t) = \delta(\mathbf{x}) \left[\lim_{N\to\infty} \frac{1}{N+1} \sum_{n=0}^{N} \delta(t+nT) \right] \, , \tag{4}$$

where T is the period of the cycle. In that case, the phase average of the sub-filtered part f' is zero, $< f' >= \overline{f'} = 0$, and the filtered, or phase-averaged quantity $< f >$ can be extracted from

$$< f(\mathbf{x}, t) >= \overline{f}(\mathbf{x}, t) = G_T * f = \lim_{N \to \infty} \frac{1}{N+1} \sum_{n=0}^{N} f(\mathbf{x}, t + nT) \; . \tag{5}$$

It is worth noting here that this filtering procedure inherently yields a decorrelation between the large scales (resolved) and the small scale (fine-grained) turbulence; whereas, in the large eddy simulation method such a cross-correlation exists.

For the most part, the large eddy simulation methodology has been based on spatial filtering. Causal time domain filters can be constructed that are analogues to the spatial filters (Pruett *et al.* 2003). The filter function in Eq. (1) is then given by Eq. (2) where $\mathcal{T}$ is the temporal filter width. It can thus be seen that within the realm of temporal filtering, it is possible to develop a more rigorous linkage between the large eddy and Reynolds-averaged approaches (Pruett *et al.* 2003), since the Reynolds-averaged function $E\{f\}$ is the limit of the temporally-filtered function $\overline{f}_{\mathcal{T}}$ when the temporal filter width $\mathcal{T}$ goes to infinity. In the frame of spatial filtering, such a linkage can only be established in homogeneous flows.

The methodologies just discussed are usually evaluated based on their predictions of integral properties of the flow or, possibly, on some time-averaged representation. rather than on higher-order statistical correlations. In addition, while details of the RANS closure or subgrid scale model may differ, the class of prediction methodology at times dictates the large scale features of the flow. Flow over a two-dimensional wall-mounted hump with blwoing/suction added for flow control is an example. Figure 1 shows time-avergaed streamline patterns obtained from experiment (Greenblatt et al. 2005), (unsteady) RANS (Rumsey 2006) and LES (Šarić et al. 2006). In the unsteady RANS predictions, one-equation, two-equation and a algebraic Reynolds stress model were evaluated (Rumsey 2006) and all yielding similar streaamline patterns. For the LES predictions, both spatial LES and DES were used and similar results were obtained (Šarić et al. 2006). These results suggest that the underlying basis for the various closures, while differing in detail, may lead to similar predictions at least at the mean variable level. These and other issues related to the prediction of non-stationary turbulent flows will be discussed.

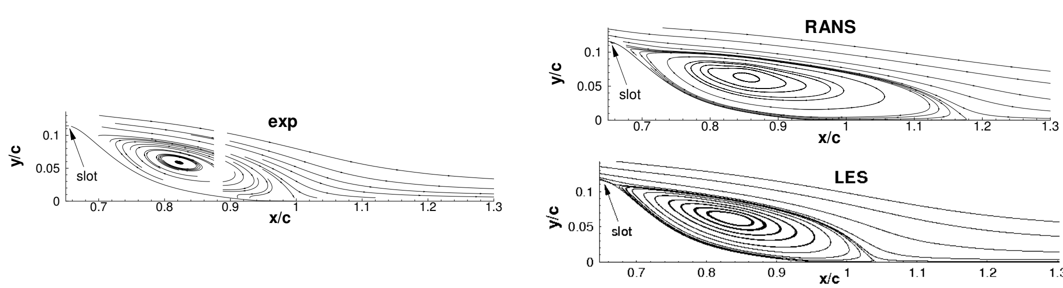

Figure 1: Long-time-average streamlines for hump model with synthetic jet flow control.

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