Global instability computations of separated flow

B. V. Bharati Laxmi and J. S. B. Gajjar

*School of Mathematics, The University of Manchester,
Manchester M13 9EP, U.K.*

Abstract. Instability of flow past a cylinder cascade is studied using linear stability analysis. A new numerical technique is used to find the critical Reynolds number when the flow becomes unstable. An attempt has also been made to study the blockage effect on critical Reynolds number and associated Strouhal number.

Key words: instability, circular cylinder, Hopf bifurcation, critical Reynolds number.

1. Introduction

The primary motivation for the current work is to develop suitable techniques for studying the global instability of separation bubbles such as those occuring in the flow past a row of circular cylinders placed in a uniform stream, see Gajjar & Azzam [3], or in the supersonic flow past a compression ramp, Fletcher et al. [1], Korolev et al. [4]. In many previous studies of the instability of separation bubbles, the basic flow is taken to be locally parallel and this is used as input into the stability analysis. Mathematically this is only appropriate when the disturbance wavelength is small compared to the distance over which the flow develops and this is not always the case. Nevertheless, in many studies conclusions stemming from such local analyses are extrapolated to generate conclusions about the global instability of the flow. This of course raises questions about the credibility of such results.
In the current work we have extended the methods used to compute the steady flow past a cascade of circular cylinders to study the instability of these flows by using a new numerical technique. Before implementing this numerical method, a number of test cases have been used for validation. One such test case is the flow in a 2-D lid-driven cavity where the global instability frequencies are found to correlate closely with temporal simulations of the linearised unsteady equations using forced disturbances. This led us to extend our techniques to study the instability of the flow past a row of circular cylinders and the details are explained in the following sections.

2. Problem formulation and implementation

The flow past a cylinder cascade is assumed to consist of an infinite number of circular cylinders placed in a uniform stream with U_∞ in x-direction, see Fig. 1. The equations governing the 2-D flow past cylinder cascade are the unsteady, incompressible Navier–Stokes equations when written in terms of streamfunction (ψ) and vorticity (ω) are given by

$$\begin{aligned} \omega_t + \psi_y \omega_x - \psi_x \omega_y &= \frac{2}{Re} \nabla^2 \omega \\ \text{and} \quad \nabla^2 \psi &= -\omega. \end{aligned}$$

Here $Re = \frac{Ud}{\nu}$ is the Reynolds number, U is the uniform speed relative to the cylinder at large distances from the cylinder, d is the diameter of the cylinder and ν is the kinematic viscosity of the fluid. All the lengthscale variables are non-dimensionalized with respect to the cylinder radius, velocities of flow with respect to U_∞ and t is the non-dimensional time. As shown in Fig. 1, the centres of the cylinder lie on y-axis i.e, at x=0 and W is the non-dimensional gap width between the cylinders. Due to

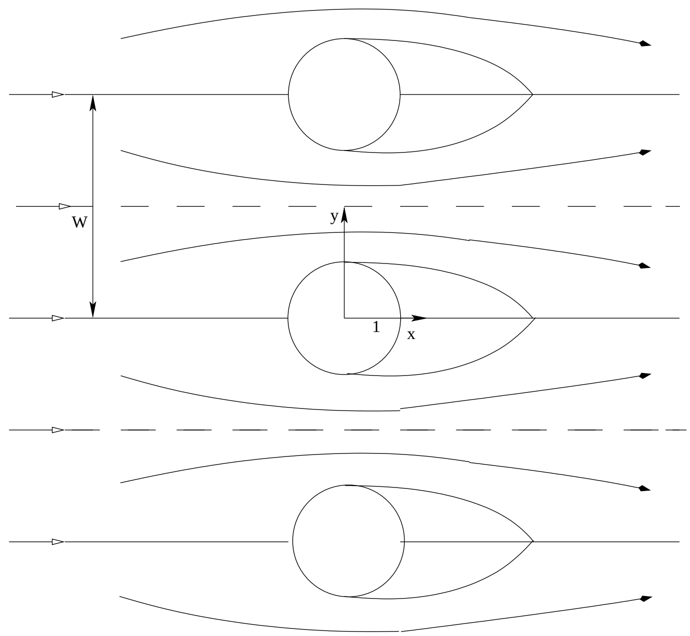

Figure 1. Sketch of flow past cylinder cascade

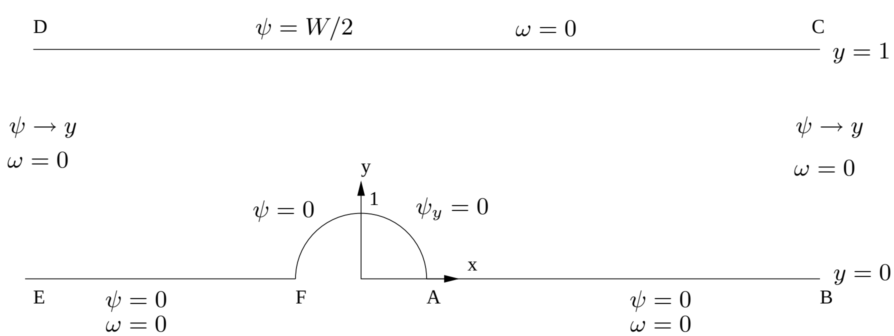

Figure 2. Sketch of physical domain

the symmetry in the flow region in Fig. 1, the problem domain is further simplified to that as shown in Fig. 2. To ease the computations, this physical domain in (x, y) plane is transformed to a strip in (ξ, η) domain by means of conformal mapping described by Fornberg [2].

Thus, the governing equations of the flow get transformed to

$$\left.\begin{aligned} \omega_t + \frac{\partial^2\omega}{\partial\xi^2} + \frac{\partial^2\omega}{\partial\eta^2} + \frac{1}{2}Re\left\{\frac{\partial\psi}{\partial\xi}\frac{\partial\omega}{\partial\eta} - \frac{\partial\psi}{\partial\eta}\frac{\partial\omega}{\partial\xi}\right\} &= 0 \\ \text{and} \quad \left\{\frac{\partial^2\psi}{\partial\xi^2} + \frac{\partial^2\psi}{\partial\eta^2}\right\}J + \omega &= 0. \end{aligned}\right\} \tag{1}$$

where $J = |dZ/dX|^2$ is the Jacobian of the transformation. These equations are solved using normal mode approach of linear stability analysis according to which

$$\psi(\xi,\eta,t) = \overline{\psi}(\xi,\eta) + \delta\widetilde{\psi}(\xi,\eta)\exp(\lambda t) \quad \text{and} \quad \omega(\xi,\eta,t) = \overline{\omega}(\xi,\eta) + \delta\widetilde{\omega}(\xi,\eta)\exp(\lambda t)$$

Substituting the above equations in (1) gives rise to a set of steady flow equations in terms of $\overline{\psi}$ and $\overline{\omega}$ and a set of stability equations in terms of $\widetilde{\psi}$, $\widetilde{\omega}$ and λ. Initially, the steady flow equations are solved for $(\overline{\psi}, \overline{\omega})$ by using a Newton-Raphson linearization technique and discretizing the resultant linearized equations. The steady flow is then substituted into the stability equations which with the same discretization gives rise to a generalized eigenvalue problem of the form

$$\mathbf{Au} = \lambda\mathbf{Bu} \tag{2}$$

wherein $\mathbf{A}$ is a large, sparse matrix with block pentadiagonal structure and $\mathbf{u} = \left(\widetilde{\psi}, \widetilde{\omega}\right)$. The generalized eigenvalue problem is solved for $\mathbf{u}$ and λ using ARPACK [6]. The numerical technique adopted for discretization is a fourth-order central difference method in ξ-direction and Chebyshev collocation method in η-direction, see [3].

3. Results

Numerical experiments were carried out on various grid sizes to check the convergence of the critical Reynolds number (Re_c) and critical Strouhal number $St_c(= \frac{fd}{U}$ where frequency, $f = \Im(\lambda)/2\pi$). This study also involved the computations with different locations of downstream boundary and varying gap widths inorder to understand their effect on Re_c. Due to space limitations, only a few important results are presented here. Table 1 shows the values of Re_c and St_c for finest grid size of each of the gap widths chosen and for a specific downstream boundary location. It

W	Grid Size ($N \times M$)	Re_c	St_c
5	64×1681	50.8	0.29519
20	81×1681	49.8	0.13584
50	101×1681	48.9	0.12108
100	101×1681	48.8	0.11806

Table 1. Re_c and St_c for varying W and grid size wherein N and M are the number of grid points in ξ and η directions respectively

is observed from the results that the flow past a cylinder loses stability to a Hopf bifurcation. The values of Re_c and St_c are found to be grid independent in each

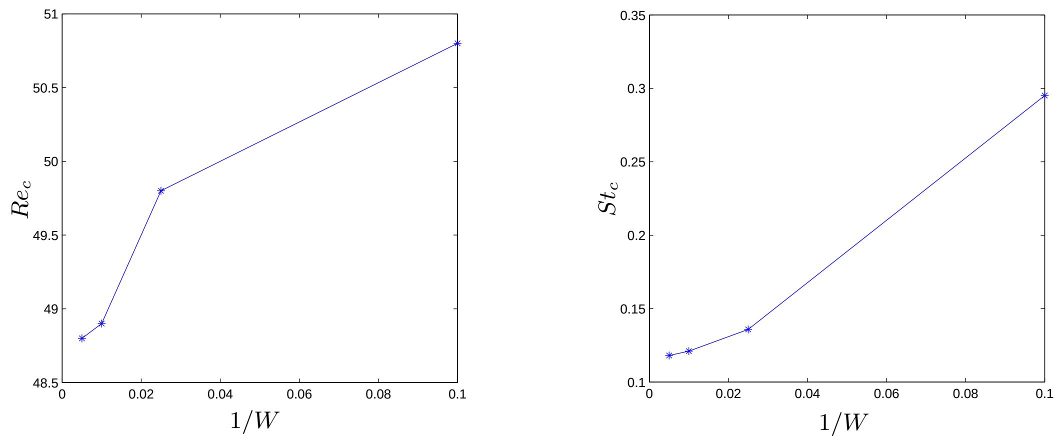

Figure 3. Blockage effect on Re_c (left) and St_c (right).

category of W as well as for different downstream boundary locations. Fig. 3 clearly illustrates that with the increase in the blockage effect $(1/W)$, there happened to be increase in Re_c as well as in St_c which is in agreement with Kumar & Mittal [5]. The contours of streamlines and vorticity and their corresponding eigenvectors for W=100 are shown in Fig. 4.

4. Conclusions

The primary instability in the wake for flow past cylinder is found to be due to a Hopf bifurcation. The critical parameters that causes the instability in the flow are found for various grid sizes in order to check the convergence. The values of Re_c and St_c are found to be 48.8 and 0.11806 for the finest grid size and for W=100. It is found that the values of critical parameters are independent of different downstream boundary locations in the case of W=5, 20 and 50 though it turned out to be computationally expensive when W=100. The values of Re_c and St_c increase considerably with decrease in W which suggests that the blockage is significant in determining when the flow loses stability.

References

[1] A J P Fletcher, A I Ruban, and J D A Walker. Instabilities in supersonic ramp flow. *J. Fluid Mech.*, 517:309–330, 2004.

[2] B Fornberg. Steady incompressible flow past a row of circular cylinders. *J. Fluid Mech.*, 225:655–671, 1991.

[3] J S B Gajjar and N Azzam. Numerical solutions of the navier-stokes equations for the flow in a cylinder cascade. *J. Fluid Mech*, 520:51–82, 2004.

[4] G L Korolev, J S B Gajjar, and A I Ruban. Once again on the supersonic flow separation near a corner. *J. Fluid Mech*, 463:172–199, 2002.

[5] Bhaskar Kumar and Sanjay Mittal. Effect of blockage on critical parameters for flow past a circular cylinder. *Int. J. Numer. Meth. Fluids*, 50:987–1001, 2006.

[6] R B Lehoucq, D C Sorensen, and C Yang. *ARPACK User's guide, Solution of large-scale eigenvalue problems with implicitly restarted Arnoldi methods.* SIAM, 1998.

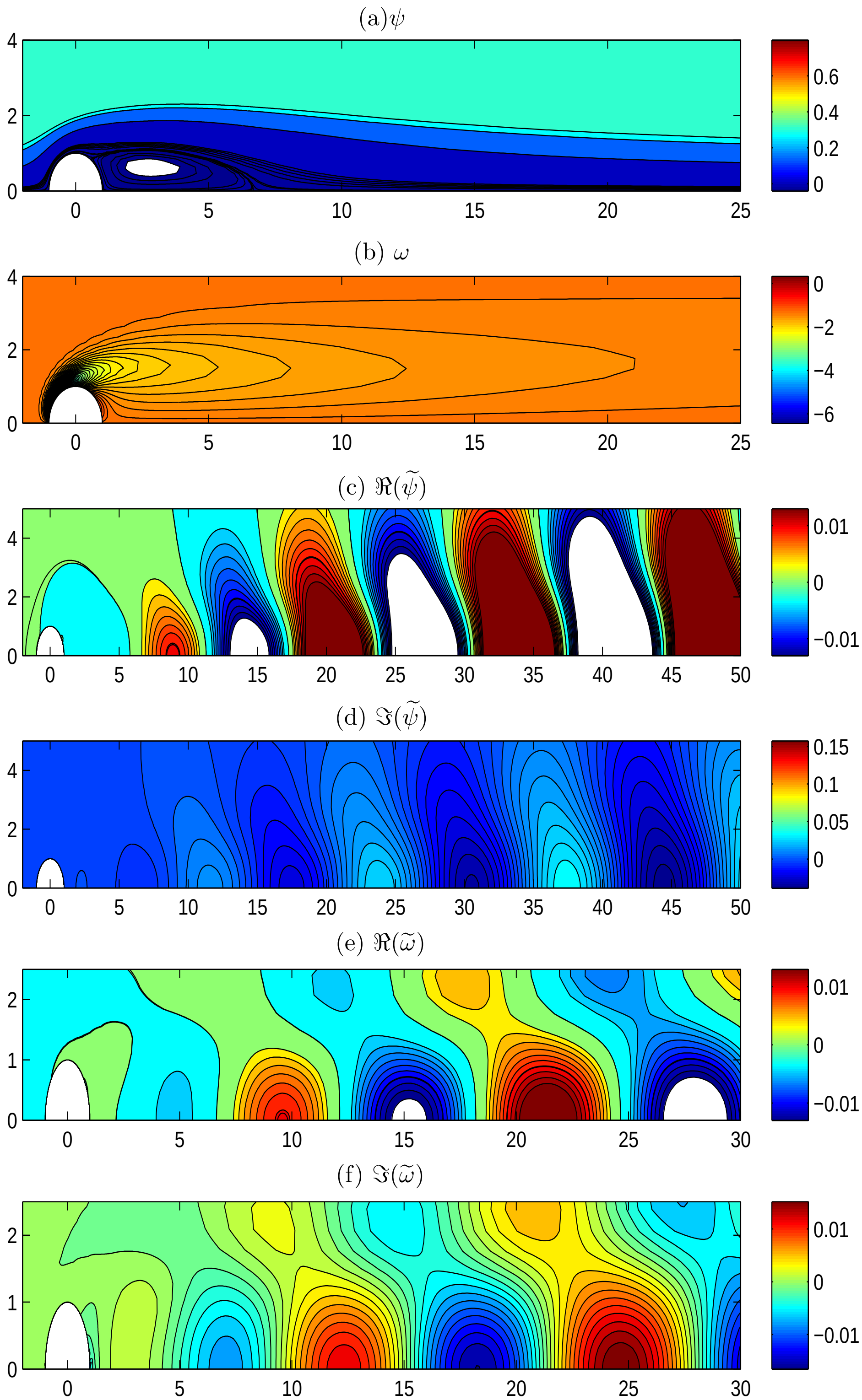

Figure 4. Contours of (a) Streamlines, (b) Vorticity, (c) real part of streamfunction (eigenvector), (d) imaginary part of streamfunction (eigenvector), (e) real part of vorticity (eigenvector) and (f) imaginary part of vorticity (eigenvector) for a grid size of M=1681, N=101 and W=100