MOTION OF A MÖBIUS BAND IN FREE FALL

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This presentation deals with the free fall of a three-dimensional object having the topology of a Möbius strip. Experimental results are shown concerning the free-fall trajectory, body motion and wake structure.

There have been scientific studies on a range of free-fall or free-rise problems, such as the free fall of plates (Dupleich 1941; Willmarth, Hawk & Harvey 1964; Smith 1971; Field *et al*. 1997; Belmonte, Eisenberg & Moses 1998; Mahadevan, Ryu & Samuel 1999; Andersen *et al.* 2005; Pesavento & Wang 2006), seed dispersal by wind (McCutchen 1977; Augspurger 1986), air bubbles or buoyant disks rising freely in a liquid (Magnaudet & Eames 2000; Wu & Gharib 2002; Fernandes *et al.* 2005). Some of the studies have looked at the vorticity dynamics and fluid forces that lead to a range of observed motions, such as zigzagging, spiralling, gyrating, tumbling and fluttering.

The present study focuses on the free-fall of a well-known body, the Möbius band. Mathematically, this shape is famous because it has only one side and one edge. The band also possesses intriguing aerodynamic properties. When placed in a uniform flow perpendicular to the plane of the centreline, the band will locally act like a thin flat plate. Due to its particular geometry, the different elements around the band will cover all possible angles of attack, positive and negative, from perpendicular to the flow to aligned with it. One therefore encounters a range of different flow situations, from flow around streamlined bodies, over high-angle of attack flows, up to bluff body wakes, all for the single object of a Möbius band. Different parts of the strip are naturally expected to experience significantly different drag forces.

If the band is not held perpendicularly to the flow, but allowed to fall freely under its own weight, it is not obvious in advance which mean orientation it will choose, since the fact that all angles of attack are present remains true, no matter from which direction the flow comes. In addition, the twisted nature of the band is likely to lead to torque forces and a resulting spinning motion, as the band moves down.

A simple physical model of a Möbius band, which is a two-dimensional non-orientable surface, can be obtained by taking a sufficiently long rectangular strip of material, twisting one end by 180°, and then gluing the two short ends together. The resulting three-dimensional object will assume a complicated shape in space, depending on the aspect ratio of the initial rectangle and the elastic properties of the material (Mahadevan & Keller 1993). There exists, however, a simple geometrical model, consisting of a circular centreline and surface elements which are locally perpendicular to this line and continuously twist around it, completing one half turn going once around the circle. A sketch of this geometry, which is defined by the diameter D of the ring and the width d of the band, is shown in figure 1.

In the present study we investigate experimentally the free fall of such an object at low Reynolds numbers.

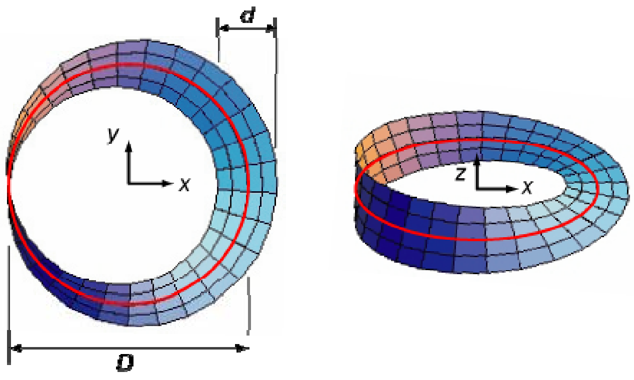

Fig. 1. Schematic of a Möbius band with a well-defined simple shape in three-dimensional space.

Fig. 2. Polyester Möbius ring (D = 45 mm) used in the experiments.

The Möbius bands were made out of sheets of polyester and polycarbonate. Since the surface depicted in figure 1 is not developable (Schwarz 1990), an approximate shape of the projection was determined empirically. The results presented here were obtained with two bands of aspect ratio $A = \pi D/d = 14$, differing by their size (D = 18 mm and 45 mm) and weight, which leads to different average sink speeds U. The larger band is shown in figure 2. The Reynolds number $Re = Ud/\nu$ is based on the sink speed, the width of the band, and the kinematic viscosity ν. For the smaller and larger ring they were Re = 130 and Re = 560, respectively.

The free-fall experiments were carried out in a water tank with glass walls of dimensions 50 cm × 50 cm × 120 cm, in which the bands were released just below the water surface. The interior of the tank was illuminated with either white light from a neon lamp placed underneath its glass base (for recordings of the ring dynamics and trajectories), or by the light from an Argon ion laser (for visualisations using fluorescent dye painted on the bands prior to release). The motion of the falling band was recorded using a digital camera, which could be displaced vertically at about the same rate as the speed of the band. The average speed of the strip was calculated from the time it took to fall from the free surface to the bottom of the tank. The frequencies characterising the time-dependent motion of the bands during their descent were obtained from analysis of the video recordings.

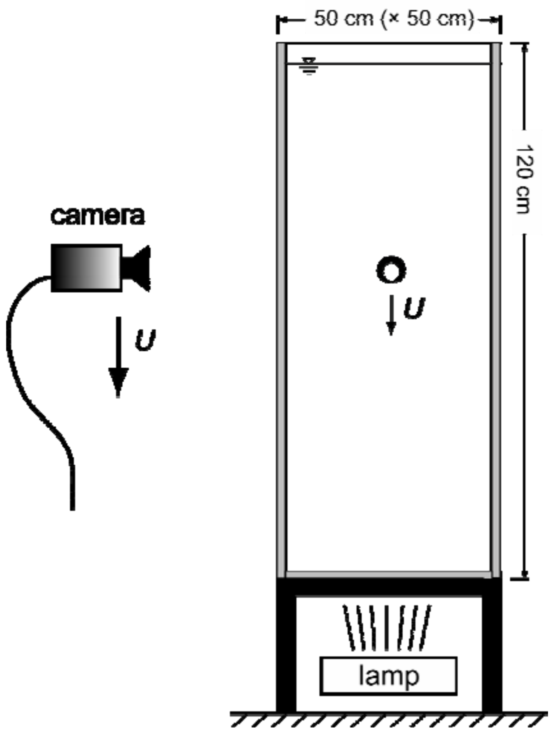

Fig. 3. Experimental setup.

The following observations were made concerning the free fall of Möbius bands at low Reynolds numbers:

- The rings orient themselves in a way that their centreline plane is almost vertical, and the blunt edge faces upstream (= downwards).
- The overall shape of the trajectory is a downward spiral with an amplitude of order *D* and a wavelength of around 10*D*, caused by the lift force on the angled surfaces at the sides of the body. While moving along this spiral path, the band spins around the vertical axis.
- The blunt leading edge induced vortex shedding which causes a vortex-induced pitching oscillation, which is superposed to the spiralling fall, at a frequency.
- Even at low Reynolds numbers, the wake structure of the falling Möbius band is extremely complicated, consisting of a system of interconnecting vortex loops and rings.

Figures 4-7 illustrate some of these observations. More details, also concerning quantitative measurements of the dynamics of the Möbius band motion, will be presented at the conference.

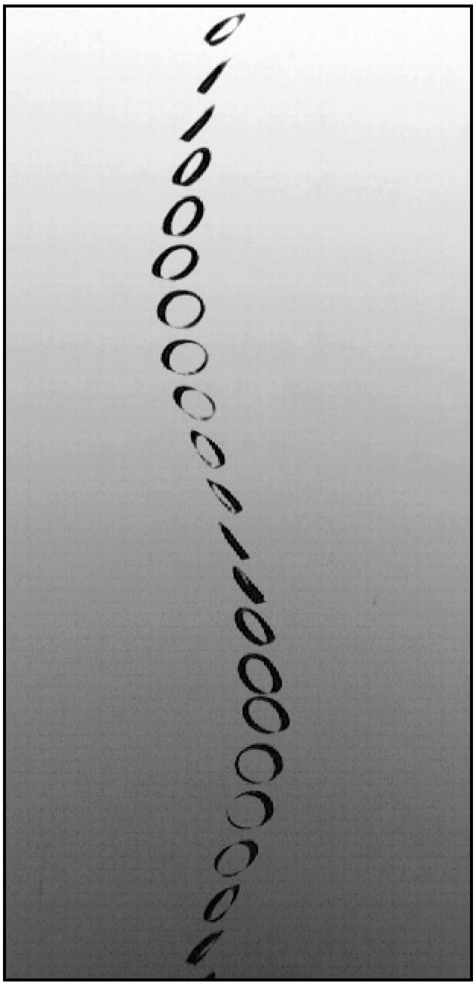

Fig. 4. Stroboscopic visualisation of Möbius band trajectory for *Re* = 130.

Fig. 5. Dye visualisation of the wake for *Re* = 130.

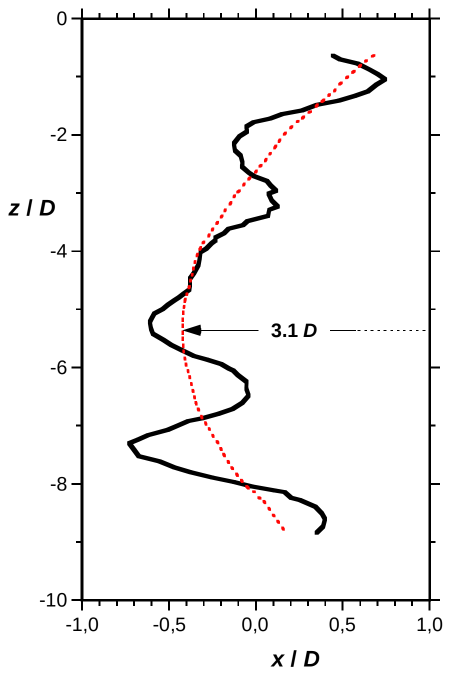

Fig. 6. Side view of the trajectory of the centre of the band for *Re* = 560. The horizontal scale (*x*) is stretched by a factor 3 w.r.t. the vertical (*z*). The red line corresponds to the overall. spiral The shedding-induced lateral oscillations are clearly seen.

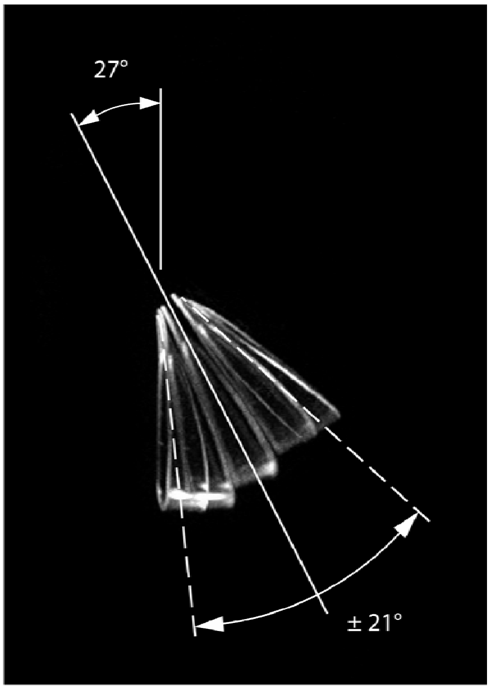

Fig. 7. Instantaneous and mean inclination of the band for *Re* = 560. Images from an "ascending" half cycle of the VIV motion were superposed after compensation for the vertical motion of the trailing edge

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