THREE-DIMENSIONAL STABILITY IN THE WAKE OF A CONFINED CIRCULAR CYLINDER, WITH TRANSVERSE OSCILLATION

Martin D. Griffith, Justin Leontini, Mark C. Thompson & Kerry Hourigan
Fluids Laboratory for Aeronautical & Industrial Research (FLAIR),
Department of Mechanical & Aerospace Engineering, Monash University, Clayton, 3800, Australia

contact email: martin.griffith@eng.monash.edu.au

A study of the two- and three-dimensional features of the low Reynolds number flow past a cylinder confined between two parallel plates is presented. For a fixed blockage ratio, the stability of the two-dimensional periodic vortex-shedding flow to three-dimensional perturbations is investigated by means of a linear Floquet stability analysis. The analysis is extended to flows in the same geometry, but with a forced transverse oscillation of the cylinder. The problem presents an interesting comparison to the unbounded case, in that the presence of the walls can significantly affect both the two- and three-dimensional features of the flow, given a high enough blockage ratio [1,3]. The present study firstly deals with the case of a stationary cylinder between two plates, before moving on to the forced transverse oscillation problem.

METHOD

For the simulations, a spectral-element solver is used to solve the incompressible Navier-Stokes equations. For the oscillatory cases, an arbitrary-Lagrangian Eulerian (ALE) mesh deformation technique is employed. The same methods have been previously employed investigating similar problems [5,6].
A schematic of the problem configuration is shown in figure 1. The geometry consists of a cylinder of

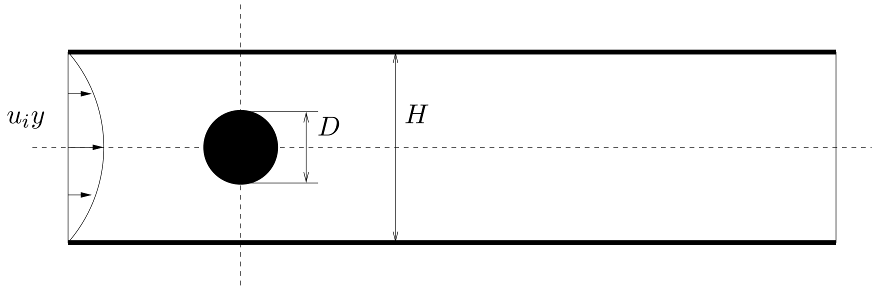

Figure 1. The problem geometry. The blockage ratio is defined as D/H.

diameter D, placed between two parallel plates spaced a distance H apart; the blockage ratio is defined by D/H. The fluid velocity profile at the inlet, $u_i(y)$ to the channel is given by a Poiseuille parabola. The Reynolds number is defined as $Re = UD/\nu$, where ν is the kinematic viscosity and U is the cross-sectional average of the inlet velocity $u_i(y)$. Note that the maximum velocity on the centreline of the channel is equal to $1.5U$. The Strouhal frequency is defined as $St = f_o D/U$, where f_o is the natural vortex shedding frequency.
Depending on the Reynolds number and blockage ratio, the well-known vortex street usually seen in cylinder wakes changes, with an inversion of the vortices occurring [1]. At a certain distance downstream, vortices shed from the upper part of the cylinder cross the centreline of the channel and are positioned in the lower half of the wake. Camarri and Gianetti, (2010) [1] argued that up to blockage ratios of 1/3, the linear three-dimensional instability modes for these "inverted" vortex streets were the same mode A and mode B instabilities as seen in the unbounded cylinder wake. They argued that the near-wake region, where the modes are strongest, is not significantly affected by the blockage; the same mode structures are hence observed. For the present study, we select a blockage ratio on this roughly-determined threshold of

1/3. This is the same blockage ratio used in Celik *et al.*, (2008) [2] in their study of mixing in the wake of a confined cylinder.

RESULTS

Figure 2(a) plots the variation of Strouhal frequency with Reynolds number for the given geometry. The

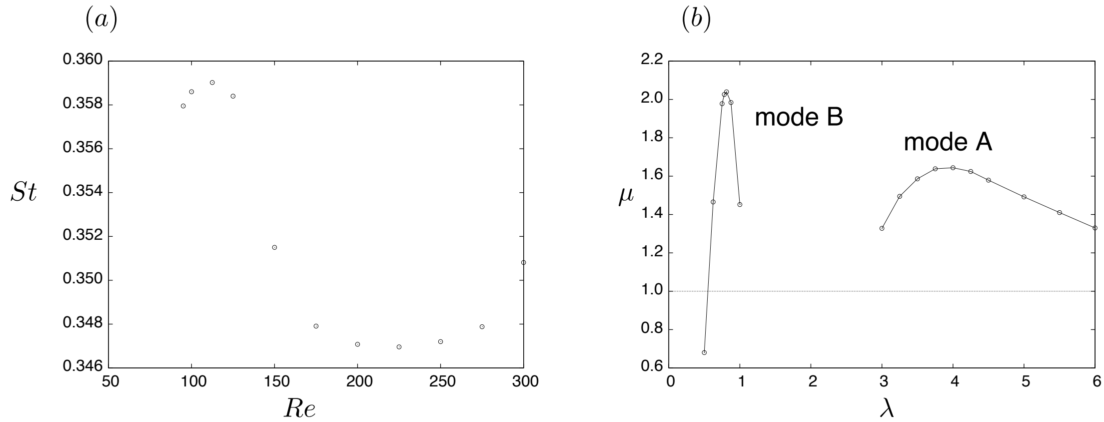

Figure 2. (a) Plot of Strouhal frequency versus Reynolds number for the stationary cylinder of blockage ratio 1/3 (b) Floquet multipliers as a function of wavelength for the case $Re = 200$.

Hopf bifurcation from a steady two-dimensional state to a two-dimensional periodic vortex-shedding state occurs for $Re \approx 98$; compared to the unbounded case ($Re \approx 46$), the transition is suppressed. The Strouhal frequencies are significantly higher than for the unbounded case. Figure 2(b) plots the results of linear Floquet stability analysis for $Re = 200$. For our analysis, a Floquet multiplier of $\mu = 1$ for a given wavelength, λ, indicates marginal stability. A multiplier greater than one indicates absolute instability. Note that for the unbounded case, the critical Reynolds number for absolute instability is $Re_c \approx 189$; the leading instability mode (mode A) is of $\lambda \approx 4.0D$ and forms in the vortex cores [4]. For the confined case, at $Re = 200$, the multipliers are comparatively much higher, indicating a critical Reynolds number much lower than found for the unbounded case. Investigation of the perturbation fields reveals that the two modes are qualitatively of the same form. In addition, the mode A instability, of $\lambda \approx 4.0D$, is no longer the leading mode. For this Reynolds number the leading mode is the shorter wavelength mode B. This mode typically becomes critical, for the unbounded case, at $Re \approx 260$ [4]. Further testing will determine a critical Reynolds number for the flow, and the leading mode.

An explanation for the increased multipliers in the confined case for mode B lies in the structure of the mode and its relation to the base flow. Typically, in the unbounded case, mode B forms in the shear layer braids between the vortex cores. This is evident in the plot in figure 3 of the $Re = 200$ base flow and perturbation field for the leading mode ($\lambda \approx 0.81$). The positioning of the mode B instability on the shear layer braids is evident. Comparing again to the unbounded case, we note that the confined case exhibits

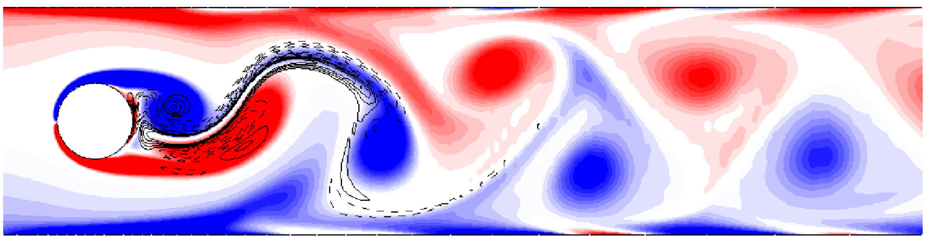

Figure 3. Contours of the base flow spanwise vorticity for $Re = 200$, overlaid with contour lines spanwise vorticity of the leading linear instability mode ($\lambda = 0.81$). Dashed lines indicate negative vorticity.

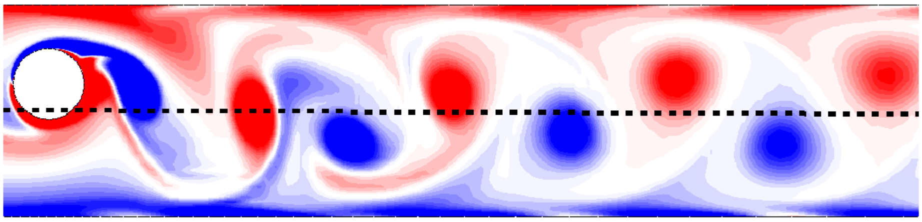

Figure 4. Contours of spanwise vorticity for the case $Re = 100$, with transverse oscillation of the cylinder, ($A = 0.4D$, $f = f_0$).

an acceleration due to constriction around the cylinder which is not present in the unbounded case. This acceleration results in a higher effective Reynolds number, which in turn creates stronger shear layers, upon which the mode B instability grows.

For the oscillatory simulations, an oscillation amplitude of $0.4D$ is used, this being the value tested in Celik *el al.*, (2008) [2]. Figure 4 plots vorticity contours for the case $Re = 100$, with an oscillation frequency $f = f_0$, with the cylinder in the upper-most position of its oscillation. The vortex shedding locks on to the natural frequency. The energy of the oscillation draws small vortices from the channel walls, which can be seen forming and then moving into the mainstream flow. For the stationary cylinder case, this phenomenon only occurs further downstream; for the oscillatory case, the near-wake region - where the linear instability modes principally act - is significantly affected.

CONCLUSION

A linear stability analysis of the flow past a cylinder confined between two plates has shown – for Reynolds numbers base on the mean inlet flow – that the mode B instability is considerably more unstable, compared to the unbounded case. This is likely due to the acceleration of the fluid through the constrictions, and the subsequent effect on the shear layers. The mode remains qualitatively the same. For the transversely-oscillating cylinder, vortices shed from the channel walls appear to penetrate further into the near-wake region of the cylinder wake. Further investigation will show how these changes in the base flow with the oscillation affect the absolute instability modes and at what limit the qualitative similarities with the modes from the unbounded case disappear.

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