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VORTEX FORCES ON AN OSCILLATING CYLINDER

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ABSTRACT

The relationship between the vortex structures in the near wake and the forces on a moving body is investigated for a cylinder undergoing forced oscillations transverse to a free-stream. Using the formulation of Wu (1981), we calculate the vortex lift force by subtracting the inertia force from the total lift force. By considering the phase of vortex shedding and the distribution of vorticity in the near wake, we find that the vortex lift force, not the total lift force, represents the force due to the vorticity field. The phase of vortex shedding does not vary with the amplitude of oscillation, and while the phase of the total lift force varies with A/D, the phase of the vortex lift force collapses towards two constant values representing two distinctly different wake states.

1. INTRODUCTION

Flow past a stationary cylinder results in large scale vortex shedding at a characteristic frequency f_o. When the cylinder oscillates at frequencies close to f_o, the perturbation of the natural instability can generate significant changes in the structure of the near wake and the forces on the cylinder. The interaction between an oscillating body and the flow field, in particular the case where the cylinder is constrained to oscillate transverse to the free-stream, has been studied extensively. The cylinders motion can be mechanically driven (forced), or the result of vortex induced forces (free).

The study of a cylinder undergoing forced oscillations by Bishop & Hassan (1963), showed that as the frequency of oscillation f_e increases through $f_e/f_o = 1$, there was an abrupt jump in the phase and amplitude of the lift force on the cylinder. The jump in the lift force appears to be a universal feature of this class of flows and has subsequently been observed over a wide range of oscillation amplitudes and Reynolds numbers: Mercier (1973), Staubli (1983), Gopalkrishnan (1993) and Sarpkaya (1995). Abrupt changes in the structure of the near wake and the mode of vortex shedding are also observed around $f_e/f_o = 1$, Williamson & Roshko (1988), Ongoren & Rockwell (1988) and Gu *et al* (1994). Subsequently, it was shown by Carberry *et al* (2001), that the jump in the amplitude and phase of the lift force corresponds to a change in the mode and phase of vortex shedding. The exact value of f_e/f_o at which this transition occurs appears to vary with Re and turbulence levels, however the mechanisms which causes these variations are not well understood. The wake states either side of the transition are referred to as the low and high frequency states. The structure of the near wake and the lift force characteristics of the low and high frequency wake states were described in detail by Carberry *et al* (2001). The general form of these wake states is strikingly similar to those observed for a freely oscillating, elastically mounted cylinder, Govardhan & Williamson (2000).

If the relationship between the flow field and the force on a body is known, then we can identify the physical changes in the wake which correspond to changes in the force on the body. The forces on a moving body can be expressed in terms of the fluid inertia and the rate of change of the vortex moment, Wu (1981):

$$\begin{aligned}\bar{F}_{total} &= -\rho\frac{d}{dt}\int_{Vc}\bar{r}\times\bar{\omega}\,dV + \rho\frac{d}{dt}\int_{Vb}\bar{u}\,dV \qquad (1)\\ &= \bar{F}_{vortex} + \bar{F}_{fi}\end{aligned}$$

where V_c is a distant external boundary containing all the shed vorticity and V_b is the volume bounding the solid body, in our case the cylinder.

The first term in equation (1) is the force due to the vorticity field, F_{vortex}, whilst the second term is the inertia force of the fluid displaced by the acceleration of the body. For a cylinder oscillating transverse to the free-stream, F_{fi} is parallel to the lift force and the fluid inertia lift force, $F_{L\,fi}$ is equal to F_{fi}. While equation (1) is a useful tool for interpreting the flow field, it has proved difficult to accurately calculate the force on a body using the vortex field within a finite two-dimensional boundary which does not contain all the shed vorticity. The idea of identifying the force component due to vorticity was also examined by Lighthill (1986). Lighthill considered two force components: a) a force due to the potential flow, which varies linearly with the velocity field and includes the potential added mass force, and b) a non-linear, vortex force. However, his vortex force represents only the force on the body due to the additional vorticity: the vorticity field minus the distribution required to generate the potential flow slip boundary condition at the body. Equation (1) is derived without simplifying assumptions and we interpret F_{fluid} without needing to assume potential flow. By coincidence, for a circular cylinder F_{fi} is equal to the potential flow added mass force for a fully submerged cylinder.

Despite the insights of Wu (1981) and Lighthill (1986) it is common to consider only the total lift on an oscillating body without considering the contributions of the vortex and fluid inertia components. In these cases the total lift force is simply referred to as the lift force. Govardhan & Williamson (2000) demonstrated that the changes in the total lift force may not fully represent the changes in the vorticity field. They calculated the vortex lift force by subtracting the "potential added mass force", which for a circular cylinder is equivalent to F_{fi}, from the total lift force. For an elastically mounted cylinder, they found that the jump in the phase of the vortex lift force did not occur at the same reduced velocity as the jump in the phase of the total lift force. The jump in the phase of the vortex lift force always corresponded to a change in the wake mode, from 2P to 2S. However, the jump in the phase of the total lift force did not necessarily correspond to a significant change in the mode of vortex shedding. In this paper we use the general equation for the vortex force (1) to describe the vortex force on an oscillating cylinder, we then relate the vortex force to the vortex structures in the near wake.

2. EXPERIMENTAL METHOD

The cylinder was forced to oscillate transversely to the free-stream such that its vertical motion was given by: $y(t) = A \sin(2\pi f_e\, t)$. Three different amplitudes of oscillation, A/D = 0.4, 0.5 & 0.6 were used, while the frequency varied from $0.74 < f_e/f_o < 1.27$. The free-stream velocity was constant at 0.090 ms^{-1} and the Reynolds number, $Re = U_\infty D/\nu$, was 2300. The cylinder, 25.4 mm in diameter, with an aspect ratio of 12.5, was fitted with end plates which oscillated with the cylinder. The velocity field of the near wake of the cylinder was measured using a laser scanning version of high-image density PIV. The images were recorded on high resolution 35 mm film and digitised at 106 pix/mm. The resulting velocity fields contained approximately 3500 velocity vectors. The phase averaged vorticity fields reported here were calculated from 9 instantaneous velocity fields. The time varying lift force on the cylinder was measured by strain gauges mounted on a support sting. The body inertia force due to the oscillation of the mass of the cylinder and the support sting, was subtracted from the lift force.

3. RESULTS AND DISCUSSION

3.1 TOTAL LIFT FORCE

For the range of oscillation frequencies and amplitudes studied the wake is "locked on" to the cylinder's motion and the sinusoidal lift force has a dominant frequency of f_e. Thus, the time varying total lift coefficient can be approximated by a sinusoidal function:

$$C_L(t) \approx C_L \sin(2\pi f_e t + \phi_{lift}) \qquad (2)$$

where C_L is the amplitude of the total lift force coefficient and ϕ_{lift} is the phase of the total lift with respect to the cylinders displacement y(t). To provide continuity with the nomenclature used in previous work the symbols C_L and ϕ_{lift} are used to describe the amplitude and phase of the total lift force.

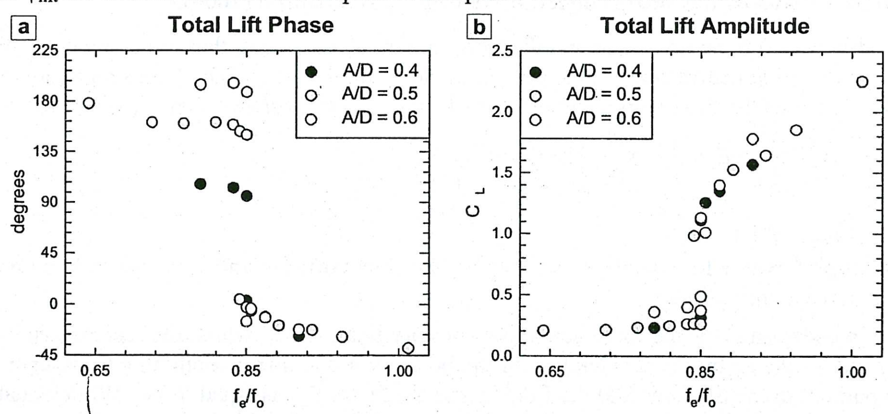


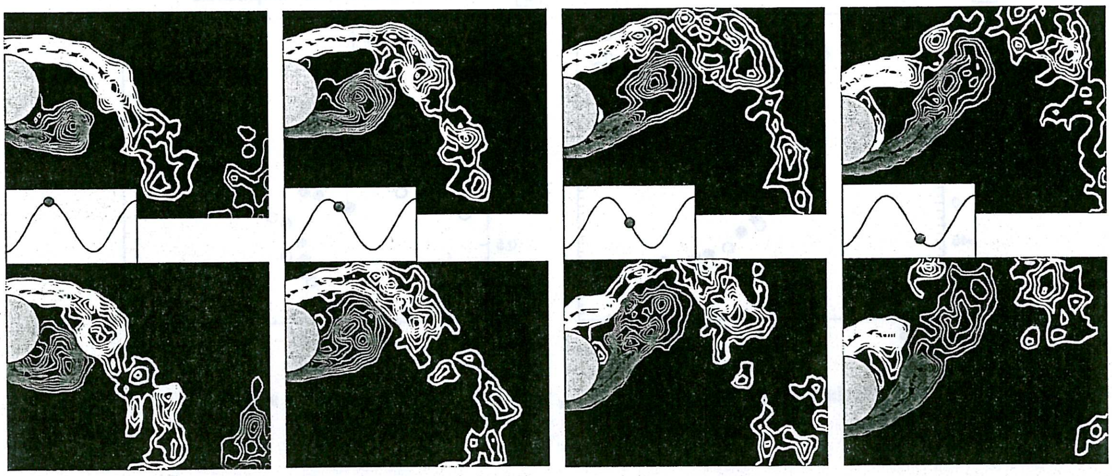
Figure 1 Variation of ϕ_{lift} and C_L with f_e/f_o, for A/D = 0.4, 0.5 and 0.6.

The variation of C_L and ϕ_{lift} with f_e/f_o, for three different amplitudes of oscillation is shown in Figure 1. For all values of A/D, at $f_e/f_o \approx 0.85$ there is an abrupt change in the amplitude and phase of the total lift coefficient, which has been shown, Carberry *et al* (2001) to correspond to a transition between two distinctly different wake states. Figure 1a shows that for the low frequency wake state ($f_e/f_o < 0.85$), there is an increase in ϕ_{lift} as the amplitude of oscillation increases. The change in ϕ_{lift} for the low frequency wake states as A/D increases from 0.4 to 0.6 is approximately 90°. However, for the high frequency wake state ϕ_{lift} appears to be independent of the changes in A/D. Similarly, in Figure 1b C_L does not vary significantly as

A/D increases from 0.4 to 0.6. It is difficult to compare the change in ϕ_{lift} at transition with the change in the phase of vortex shedding, as transition also corresponds to a change in the mode of vortex shedding. However, the variation of ϕ_{lift} with A/D for the low frequency wake mode, allows us to compare changes in ϕ_{lift} with the phase of vortex shedding for a given mode of vortex shedding.

Figure 2 shows the time evolution of the low frequency wake state for A/D = 0.4 and 0.6. Despite the change in the amplitude of oscillation, the mode and timing of vortex shedding are very similar. However, it is clear that as is A/D increased from 0.4 to 0.6, the vertical movement of the wake increases, and there is a change in the distribution of vorticity in the near wake. The vorticity fields in Figure 2 correspond to the two data points at f_e/f_o = 0.823 in Figure 1. Despite the similarity in the phase of vortex shedding in Figure 2, the values of ϕ_{lift} are very different, 103° and 196° for A/D = 0.4 and 0.6 respectively Thus, it is evident that the phase of the total lift force (total) does not represent the phase of vortex shedding.

A/D = 0.4, f_e/f_o = 0.823



A/D = 0.6, f_e/f_o = 0.823

Figure 2 Phase averaged vortex structures for the low frequency wake state as the cylinder moves through its downwards stroke. The amplitude of oscillation in the upper row is A/D = 0.4, while the lower row shows the corresponding images at A/D = 0.6. The inserts indicate the displacement of the cylinder.

3.2 VORTEX LIFT FORCE

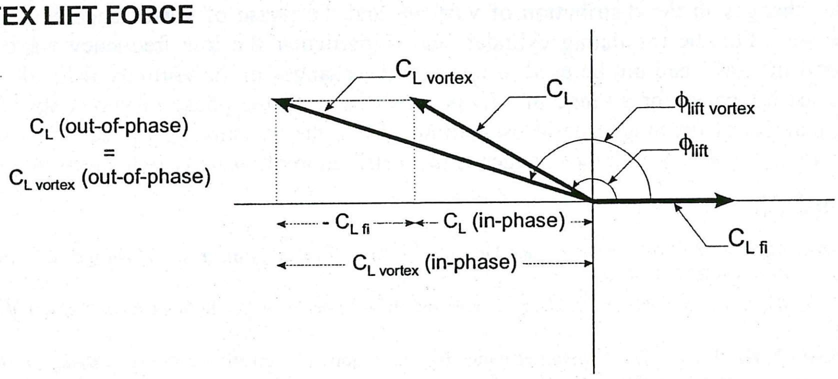


Figure 3 Vector diagram showing the relationship between $C_L(t)$ and $C_{L\ vortex}(t)$.

The vortex lift force is calculated using equation (1) by subtracting the fluid inertia force F_{fi} from the total lift force. The fluid inertia, $F_{fi} = -\rho \ Vol \ d^2y(t)/dt^2$, is purely in-phase with y(t) and subtracting F_{fi} does not alter the out-of-phase component of the lift force. The magnitude of F_{fi} increases with the amplitude of oscillation, therefore as A/D increases from 0.4 to 0.6 there is a 50% increase in the magnitude of F_{fi}. The vector relationship between $C_L(t)$ and $C_{L\ vortex}(t)$ in Figure 3, demonstrates that the subtraction of F_{fi} affects both the phase and amplitude of the lift force. The vortex lift force is now expressed in the same way as the total lift force:

$$C_{L\ vortex}(t) \approx C_{L\ vortex} \sin(2\pi f_e t + \phi_{lift\ vortex}) \qquad (3)$$

The variation of the phase and amplitude of the vortex lift force (Figure 4) is significantly different from the phase and amplitude of the total lift force (Figure 1). For both the low and high frequency wake states there is a collapse of $\phi_{lift\ vortex}$ for all values of A/D. For the low frequency wake state $\phi_{lift\ vortex}$ collapses towards

180^o, while for the high frequency wake mode the vortex lift phase tends towards -90^o. Unlike C_L, $C_{L\ vortex}$ increases with f_e/f_o for both the low and high frequency wake modes and at transition $C_{L\ vortex}$ jumps downwards. For the low frequency wake state $C_{L\ vortex}$ increases with A/D, while for the high frequency wake state $C_{L\ vortex}$ does not vary significantly with A/D. Therefore, the magnitude of the jump in $C_{L\ vortex}$ at transition varies with A/D.

The collapse of $\phi_{lift\ vortex}$ indicates that for both wake states the phase of vortex shedding is approximately independent of A/D. This is consistent with the timing of the vortex shedding for the low frequency wake state in Figure 2. The vorticity fields for the high frequency wake state (not shown) also show that the phase of vortex shedding does not vary with A/D. According to equation (1) a change in the distribution of vorticity will alter the magnitude of the lift force on the cylinder. Thus the variation of $C_{L\ vortex}$ with A/D for the low frequency wake state, is consistent with the change in the distribution of vorticity observed in Figure 2.

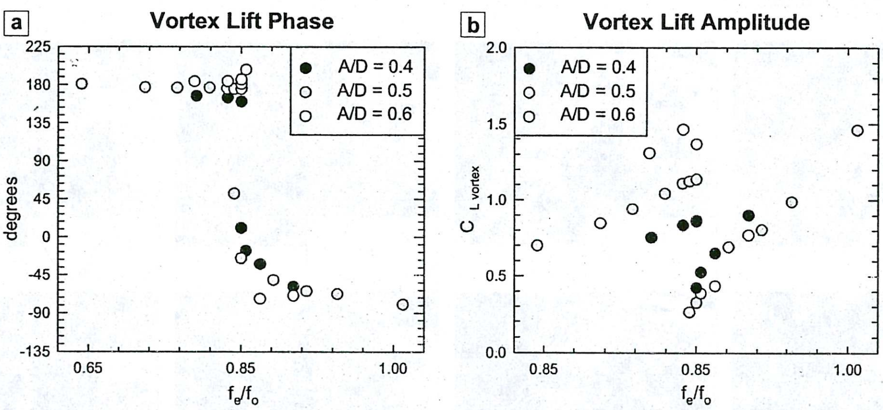


Figure 4 Variation of $\phi_{lift\ vortex}$ and $C_{L\ vortex}$ with f_e/f_o, for A/D = 0.4, 0.5 and 0.6.

4. CONCLUSION

In this paper we investigate the relationship between the lift forces on an oscillating cylinder and the structure of the near wake. The total force on a moving body includes a component from the inertia of the displaced fluid, F_{fi} and a component due to the rate of change of the vorticity field, F_{vortex}. The lift forces on a cylinder oscillating transverse to the free-stream were examined for a range of oscillation amplitudes and frequencies. To determine the difference between the total force and the vortex force we compare the changes in $C_L(t)$ and $C_{L\ vortex}(t)$ with the changes in the distribution of vorticity and the phase of vortex shedding, as A/D and therefore F_{fi} increases. For the oscillating cylinder, and in particular the low frequency wake state, it is evident that the total lift force can not be used to interpret the changes in the vorticity field. However, the collapse of the vortex lift phase for a range of A/D is consistent with the phase of vortex shedding, which appears to be independent of the amplitude of oscillation. While the variation of $C_{L\ vortex}$ with A/D for low frequency wake state, is consistent with the changes in the distribution of vorticity in the near wake.

5. REFERENCES

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