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Flow Induced Acoustic Resonances for a Bluff Body in a Duct: A Numerical Study

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ABSTRACT

The generation of an acoustic resonance is modelled for flow past a bluff body in a duct. This paper examines the situation where the rate of vortex shedding is locked to the resonant acoustic frequency. The bluff body has round leading and round trailing edges. The resonant acoustic mode in the duct is predicted via a finite-element solution of the Helmholtz equation; good agreement with acoustic frequency is found. The vortex shedding from the plate is predicted using a surface vorticity model of the plate surface and a discrete-vortex model of the shed vorticity. The transfer of acoustic power to the sound field from the flow is predicted using Howe's integral. It is found that an imbalance in power transfer to the acoustic field exists during the acoustic cycle a vortex is formed; the region at the trailing edge of the plate therefore represents an acoustic source.

NOMENCLATURE

A	matrix of coefficients
B	amplitude of acoustic particle velocity
c	speed of sound
H	thickness of the plate
I	identity matrix
K	matrix of coupling coefficients for surface elements
$\underline{n}$	unit normal vector to body surface
p	pressure
P	rate of power transfer to acoustic field
Q	matrix of coefficients coupling free vortices to surface elements
s	distance along body surface
St	$= \Omega H/(2\pi V)$ acoustic Strouhal number
t	time
T	acoustic period
$\underline{u}$	acoustic particle velocity
$\underline{v}$	fluid velocity
v_x	component of fluid velocity in x direction
v_y	component of fluid velocity in y direction
$\underline{V}$	flow velocity at upstream infinity
x	horizontal coordinate
y	vertical coordinate
Z	number of free elemental vortices in the flow

Greek symbols

α	angle between acoustic particle velocity and vortex velocity
β	simplest acoustic mode for a plate in a duct
γ	surface vorticity density
δ	Kronecker delta
Δs	length of surface segment
λ	eigenvalue
ϕ	solution of the Helmholtz equation
$\underline{\nabla}$	divergence operator
$\underline{\omega}$	vorticity
Ω	acoustic angular frequency

INTRODUCTION

It is well established that flows around bluff bodies in ducts can lead to acoustic resonances which can feed back and control the rate of vortex shedding (e.g. Welsh, Stokes and Parker, 1984). Since it is the regions of rotational flow - the vortices - that are the sources (or sinks) of acoustic power (Howe, 1975), changing the vortex shedding rate can lead to increased levels of acoustic energy in a duct. A constant amplitude sound field results when equilibrium is reached between the acoustic generation by vortices and loss of energy due to mechanical damping and losses at the openings of the duct (Welsh et al., 1984). The simplest example of a flow pattern together with an acoustic resonance - a single plate and a Parker β-mode - has been examined in some detail experimentally (Cumpsty and Whitehead, 1971; Archibald, 1975; Welsh and Gibson, 1979; Welsh, Stokes and Parker, 1984; Stokes and Welsh, 1986). The papers by Welsh and Stokes also invoke Howe's (1975) theory of aerodynamic sound to provide a theoretical description of the mechanism of excitation of an acoustic resonance.

This paper presents a more sophisticated numerical method for investigating the generation of acoustic resonances for a plate in a rigid walled duct. The surface vorticity method of Lewis (1981) is used to simulate the potential flow around a bluff body and the shed vorticity is represented by discrete vortices. The discrete-vortex model has been used to model a variety of separated flows (e.g. Clements, 1973) and has been suggested previously as a technique for investigating flow/acoustic interaction (Rockwell and Naudascher, 1979). An acoustic field, predicted by a finite-element calculation, is superimposed on the flow field; the simplest mode, the Parker β-mode (Parker, 1966), being selected. The intensity of the acoustic field is chosen to be sufficiently large to 'lock' the vortex shedding rate to the acoustic frequency. The prediction of the output acoustic power caused by a vortex interacting with the acoustic field is made using the formulation of Howe (1975). One of the advantages of the present approach is that the shear layers leaving the body are better represented and the phase at which large-scale structures are released is determined. The method described below can also be extended to model more complex flows around bluff bodies of arbitrary shape and number.

DESCRIPTION OF THE MODEL

a. The acoustic field

The plate and duct geomet considered are: plate chord, 8.375H; duct lengtn, 347.125H; duct width, 30.5H. The plate is positioned in the centre of the duct. [illegible]se dimensions are chosen to correspond to those currently being used in experiments at the CSIRO Division of Energy Technology, where H is typically 8 mm.

The acoustic modes observed are simple harmonic oscillations in time. The time variable may then be eliminated from the wave equation by solving for the modified variable ϕ defined by $p = \phi e^{i\Omega t}$. ϕ then satisfies the Helmholtz equation

$$\nabla^2\phi + (\Omega/c)^2 \phi = 0 \quad (1)$$

The boundary conditions for the transverse modes are

$$\underline{n} \cdot \underline{\nabla}\phi = 0 \quad (2)$$

on rigid surfaces ($\underline{n}$ = surface normal vector) and

$$\phi = 0 \quad (3)$$

on the duct midline (except that part occupied by the plate).

This elliptic boundary value problem is solved by a finite element method. Triangular elements with piecewise linear shape functions are used. The resulting system of linear algebraic equations for the values of ϕ at the nodes is symmetric and banded. The mode frequencies are the eigenvalues of the matrix of coefficients. The corresponding eigenvectors give the nodal pressure amplitudes. The smallest eigenvalue gives the simplest solution, for which ϕ has the same sign in the solution region (one half of the duct). For a single plate, this is the Parker β-mode (Parker, 1966).

If A is the symmetric banded matrix of coefficients, then the eigenvalue λ is found by solving

$$\det(A - \lambda I) = 0 \quad (4)$$

by the secant method.

b. The flow field

The surface of a bluff body may be represented by a vortex sheet. The distribution of vorticity along the surface must meet the requirement of zero velocity on the inside of the sheet and ensure that the contour is a streamline. In the technique put forward by Lewis (1981), the linear vorticity density $\gamma(s)$ along the surface of a two-dimensional body in irrotational flow is provided by the solution to the Fredholm equation. The appropriate equation for the present flow is:

$$\oint\gamma(s_n)K(s_n,s_m)ds_n - 0.5\gamma(s_m) = -v_x(dx_m/ds)-v_y(dy_m/ds) \quad (5)$$

The coupling coefficient $K(s_n,s_m)$ is the surface tangential velocity at s_m induced by a unit vortex at s_n.

Discretizing the body profile into M segments of length Δs produces a set of linear equations

$$\sum_{n=1}^{M} \gamma(s_n)\Delta s_n(K(s_n,s_m)-0.5\delta_{n,m}) = -v_x(dx_m/ds)-v_y(dy_m/ds)$$

$$-(1/2\pi)\sum_{n=1}^{Z} \Gamma_n Q(n,m), \quad (m=1,M) \quad (6)$$

where the last term in equation (6) gives the contribution to the velocity field at the surface due to Z free vortices in the flow. The solution of equation (6) gives the surface vorticity density at the pivotal points on the surface, which are taken to be the centre of each discrete element.

c. Generation and Shedding of Vorticity

Vorticity is generated at the body surface due to tangential pressure gradients (Lewis, 1981; Morton, 1984). For an inviscid fluid, the boundary layer thickness is infinitesimal and is represented by the surface vorticity elements. The velocity at the outer edge of the boundary layer is equal to the surface vorticity density. In the present model, the vorticity shedding rate is calculated using the surface vorticity density and convection velocity at the centre of the surface segments immediately preceding the rounded trailing edge. No vortices are assumed to shed from the round leading edge.

Interaction between the point vortices is 'smoothed' by assuming a Rankine vortex velocity field within 0.05H of their centres. The vortices are advected using a second-order Euler scheme. A time-step of 0.05 (H/V) is used and new vortices are released every fourth time-step.

d. The excitation of sound by vorticity

According to the theory of aerodynamic sound of Howe (1975), the rate P at which a vortex of vorticity $\underline{\omega}$ moving with velocity $\underline{v}$ does work on a sound field with local acoustic particle velocity u is given by

$$P = |\underline{\omega}|\ |\underline{v}|\ |\underline{u}|\sin\alpha \quad (7)$$

Here, the vorticity is assumed to have a direction perpendicular to the two dimensional flow and acoustic fields. α is the angle between the flow and acoustic particle velocities.

In order that a net positive power be transferred to the acoustic field, an imbalance in P over an acoustic cycle must occur on some occasion. In the present case, a constant acoustic amplitude is assumed in line with observation. The imbalance must therefore arise out of varying strength, varying velocity amplitude or varying velocity direction of a vortex. These are conditions that can occur during the growth and release stage of a vortex structure (Welsh et al., 1984).

In the following results, the acoustic Strouhal number is taken to be 0.21, corresponding approximately to the natural vortex shedding frequency observed for the present plate geometry. The amplitude of the acoustic particle velocity at the centre of the trailing edge of the plate is set at 0.25V, consistent with the values observed for strong resonant fields.

RESULTS

The simplest resonant transverse duct acoustic mode with one plate located on the duct centreline is the β-mode as defined by Parker (1966). The resonant frequency for this mode is predicted to be lower by 2.4% than for the corresponding mode when no plate was present, in line with current observations at the CSIRO Division of Energy Technology. The acoustic particle velocities corresponding to the β-mode are shown in Figure 1 when the velocities are in the upwards direction. The mode frequency is below 'cut-on' of the first cross mode of the duct without any plates installed. Consequently, there is exponential decay in the amplitude of the acoustic particle velocities in both the upstream and the downstream directions away from the plate.

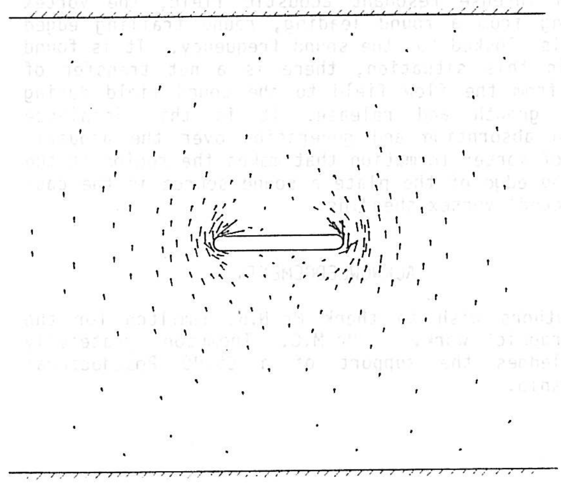

Fig 1: Relative acoustic particle velocities

The release of large-scale vortex structures from the trailing edge of the plate is found to be 'locked' to the acoustic frequency. During each acoustic cycle, a new vortex is formed at each corner of the trailing edge of the plate. The vortices at opposite corners are formed at half an acoustic cycle apart. Instantaneous 'snapshots' of the vortex positions over approximately an acoustic cycle are shown in Figure 2; the first diagram showing a new vortex being formed from the top corner of the plate. The phase angle at which a large scale vortex begins to form corresponds approximately to the time when the acoustic particle velocity changes direction towards the opposite corner. Similar structures, shown in Figure 3 for an acoustic phase angle comparable to Figure 2b, are observed experimentally from smoke visualization (Welsh et al., 1984); these authors observed the vortices forming at a phase angle similar to that shown by the predictions.

The acoustic power output for a large-scale vortex structure during its formation over an acoustic cycle is shown in Figure 4. The imbalance in power output between the first half cycle and the second half cycle is clearly evident. When a new vortex forms, there is a net positive transfer of energy to the acoustic field over the acoustic cycle. The trailing edge of the plate therefore becomes an acoustic source region. The following factors contribute to the imbalance: (1) the average strength $|\underline{\omega}|$ of a vortex is lower during the first half cycle of its growth; (2) the magnitude of the velocity $|\underline{v}|$ of a vortex as it rolls up from the separating shear layer is lower in the first half cycle; the angle α between the velocity of a growing vortex and the acoustic particle velocity appears to be lower near the edge of the plate than further downstream where it is approximately 90°. Each of these factors lead to a smaller magnitude of P in the Howe equation (7). These factors evidently outweigh the higher amplitude acoustic particle velocities found close to the edge of the plate leading to a higher $|\underline{u}|$ in the Howe equation.

The flow is periodic and there is a symmetrical phase relationship between the growing vortex and the acoustic field on each side of the plate. This means that all vortices transfer a net positive power to the sound field near the trailing edge. Comparison with the corresponding results of a more idealized prediction by Welsh et al. (1984) shows very good correspondence with the present relative minimum and maximum instantaneous power outputs during this first cycle.

CONCLUSIONS

The sound field in a duct is modified by the presence of a plate. In the present case, the resonant frequency is lowered by 2.4% for the simplest mode, the Parker β-mode. The acoustic particle velocity amplitude decays exponentially away from the plate.

For an intense resonant acoustic field, the vortex shedding from a round leading, round trailing edged plate is locked to the sound frequency. It is found that in this situation, there is a net transfer of power from the flow field to the sound field during vortex growth and release. It is this imbalance between absorption and generation over the acoustic cycle of vortex formation that makes the region at the trailing edge of the plate a sound source in the case of 'locked' vortex shedding.

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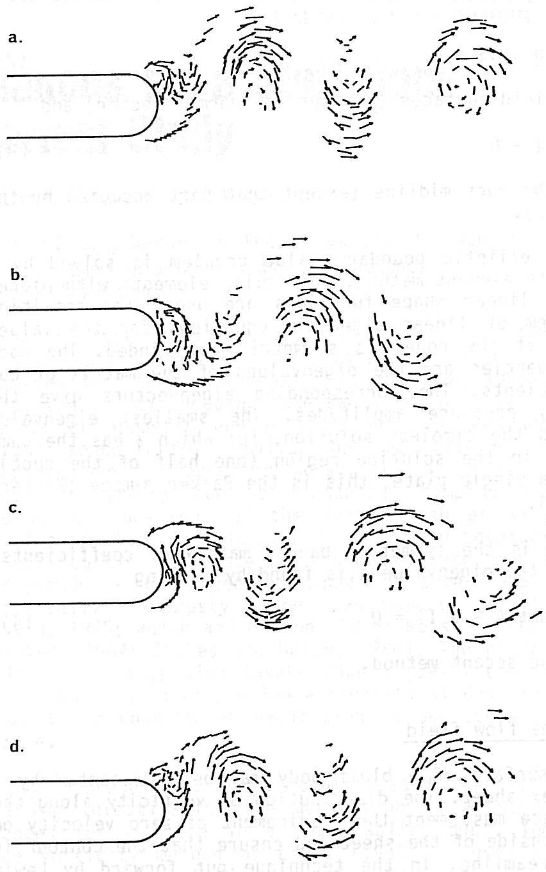

Fig 2: Plot of relative velocities of the elemental vortices during growth at acoustic phase angles (in degrees relative to zero velocity increasing up): a. 79, b.180, c.295, d.396.

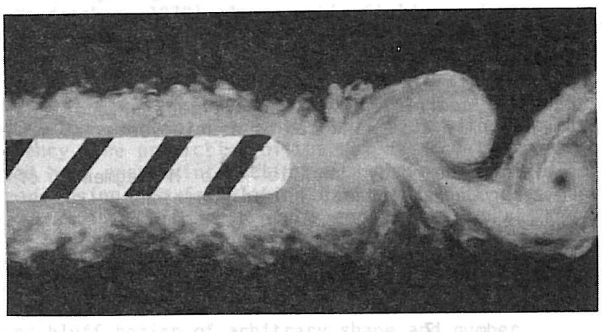

Fig 3: Flow visualisation of observed wake at acoustic phase angle comparable to Fig 2b.

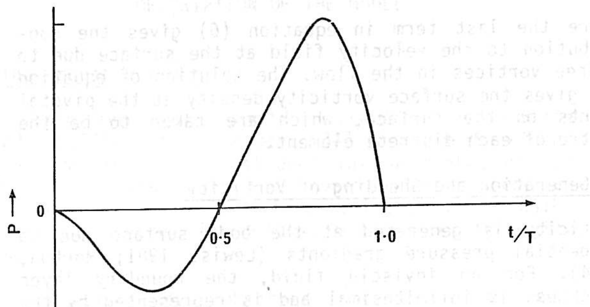

Fig 4: Plot of power P generation due to a single large scale vortex structure over an acoustic cycle during formation

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