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CHARACTERISATION OF THE SEPARATED SHEAR LAYER IN THE NEAR-WAKE OF A CIRCULAR CYLINDER

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ABSTRACT

The objective of the present study to characterise the separated shear layer in the near-wake of a circular cylinder is to provide a 'base case' for comparison with future investigations on how the stability character changes under different flow regimes - namely different turbulence levels, and also the application of acoustic perturbation.

Measured velocity profiles showed that the wake is convectively unstable up to $x/D = 0.66$. The transition point branch-point frequencies predicted by linear stability analysis agreed well with measured ones. The study also indicated that the shear layer instability waves can only be triggered by sufficiently freestream turbulence.

INTRODUCTION

The application of linear stability analysis in shear flows experienced major advances in recent years. Huerre and Monkewitz have been notable, among other researchers, in leading the application of the concept of absolute and convective instabilities to further characterise the instability of shear flows (see for example Huerre & Monkewitz, 1985). The goal of such a descriptive scheme is to determine for any flow configuration the applicability of spatial stability theory, where the frequency ω is real and the wavenumber k is complex (Huerre & Monkewitz, 1985).

The impulse response of a flow is defined as the instability-wave field generated by a concentrated pulse in space and time. A flow is then absolutely unstable if its asymptotic (large time) impulse response becomes unbounded in space (see fig. 1a). The flow is convectively unstable if the asymptotic impulse response decays to zero at all spatial points (see fig. 1b). In absolute unstable flows, the presence of a transient disturbance at any point leads, in the linear regime (small disturbance), to exponential growth everywhere. In convectively unstable flows disturbances are convected away as they amplify, leaving an undisturbed basic flow (Huerre & Monkewitz, 1985).

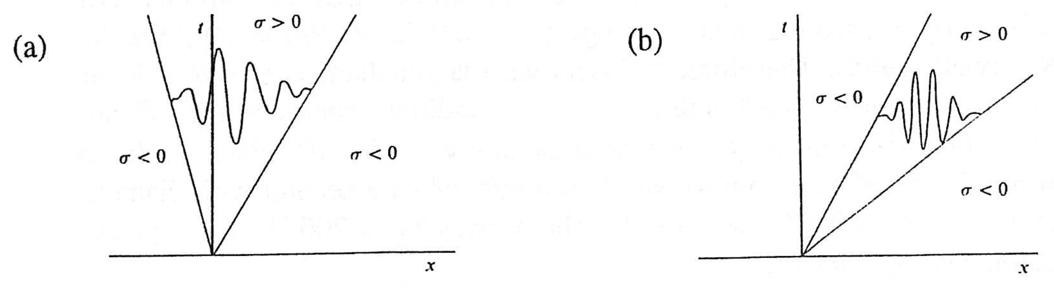


Figure 1. Sketch of typical impulse response: (a) Absolutely unstable flows; (b) Convectively unstable flows. After Huerre & Monkewitz (1985).

The main physical implication is the response of the flow to spatially concentrated, constant amplitude harmonic source which is described by spatial theory. In a convectively unstable flow, this response is observable because any transient generated by the switching-on of the source and background noise is convected away and dies off at any downstream station. Thus, any variation in the source is reflected precisely in the instability wave after the associated transients has died away. Therefore the response of such flows is sensitive to the nature of the forcing of any flow disturbances. It can be viewed as an amplifier with an infinite signal-to-noise ratio (SNR) in the limits of large time. For an absolutely unstable flow, the switch-on transient grows in space and eventually overwhelms the spatial response to the localised harmonic source. Thus response is swamped by noise at all spatial points of the flow. It can be viewed as an oscillator with a zero SNR in the limit of large time (Monkewitz & Nguyen, 1987).

Linear stability analysis has shown that the onset of the Kármán vortex formation in two-dimensional wakes at low Reynolds numbers (*Re*) is due to the absolute instability of the wake (Chomaz *et al.*, 1988; Monkewitz, 1988)

At high *Re*, bluff-body wakes are different from mixing layers (where the dominant frequency is predicted by the maximum amplification rates calculated from stability analyses) in that they are dominated by the Kármán vortex frequency. The selection of the dominant wake mode therefore requires a global criterion. Betchov & Criminale (1966) first proposed that the response of the flow is dominated by the resonance between a downstream and an upstream travelling instability wave. The amplification or damping of this resonance is related to their absolute or convective nature. The conditions for resonance is determined locally and this varies with streamwise position.

Koch (1985) and Monkewitz & Nguyen (1987) proposed different selection criteria and applied them to idealised mean velocity profiles of wakes. They reported good agreement with experimental evidence. Further analysis by Monkewitz & Nguyen (1987) suggests that the applicability of each criteria is dependent on the degree of wake-body interaction and therefore the shape of the profile used. Gorman *et al.* (1992) reported good agreement between measured vortex frequency and that predicted using Koch's (1985) criterion applied to velocity profiles for an airfoil generated by Direct Numerical Simulation.

Another feature of interest at high *Re* is the high frequency shear layer vortices which was first reported by Bloor (1964). By the application of boundary-layer theory, Bloor deduced a *Re* power-law relationship between the frequency of the Kármán vortices and the shear layer vortices, such that $f_{SL}/f_K \propto Re^{0.5}$. Using a flow visualisation technique, Wei & Smith (1986) found, contrary to Bloor, that the shear layer vortices follow a 0.87 power law. They also showed that these vortices do not extend across the whole span, but form cellular spanwise structures. Subsequent studies suggested that the 0.5 power law to be more appropriate (Kourta *et al.*, 1987; Filler *et al.*, 1991; Wu, 1992).

These scaling laws nevertheless has a problem. It is well known that the Strouhal No. ($St = f_K D/U$) remains fairly constant in this *Re* range ($St \sim 0.2$ for $O(Re) = 10^3$. See for example Unal & Rockwell, 1988). Therefore, cylinders with larger diameters have a lower f_K. Cylinders of different diameters placed in different flows conditions such that their *Re* are the same, may end up with vastly different f_{SL}. Eg. In air, where $\nu = 1.5 \times 10^{-5}$ Pa·s, a cylinder of diameter $D_1 = 6$ mm is placed in a flow where $U_1 = 6$ m/s, while a second with diameter $D_2 = 12$ mm in a flow where $U_2 = 3$ m/s. $Re = 2400$ for both cases. $f_{K1} = 200$ Hz, $f_{K2} = 50$ Hz. Using whichever scaling law, $f_{SL1} \neq f_{SL2}$.

The calculations by Monkewitz & Nguyen (1987) for ideal wake profiles indicated a spatial amplification maximum at frequencies higher than the Kármán vortex frequency, and these could be interpreted as the "mixing layer mode" instabilities on the shear layers which roll up to form the Kármán vortices. Michalke (1965) has already shown that for the generic free shear layer, the most amplified frequency f_M, is related to the mean shear-layer velocity U_{av}, and momentum thickness θ by $f_M\theta/U_{av} = 0.032$, and this relationship has been experimentally tested with considerable success (Ho & Huerre, 1984). Filler *et al.* (1991) showed that this relationship is satisfied in the case of the separated shear layer from a circular cylinder under rotary motion by approximating θ with $D/Re^{0.5}$ and U_{av} with $0.70U$.

Wu (1992) investigated the effects of acoustic perturbations on the circular cylinder wake and found that the Kármán vortex frequency is changed by as much as 10%. The nonlinear interaction between the shear layer and the Kármán instabilities are much more pronounced in the presence of acoustic perturbation.

The present work attempts to characterise the separated shear layer in the near-wake of a circular cylinder by measuring the velocity profiles and the growth of the velocity perturbation. Precise measurement of the profile is obtained through a high resolution stepper-motor driven traverser. At the *Re* range to be investigated, it is anticipated that the recirculation zone is sufficiently far downstream of the rear-facing stagnation point, so as not to present a problem with velocity measurements by hot-wire anemometry.

By fitting idealised velocity profiles to the measured profiles, stability analysis is performed to investigate the ability of linear stability theory to predict the shear layer frequencies and hence provide an understanding of how the shear layer instability mode develops and evolves.

EXPERIMENTAL DETAILS

The experiments were conducted in an open-circuit wind tunnel. The working section of the tunnel has a 244 mm × 244 mm cross section. The tunnel turbulence level is very low. When first characterised, the turbulence level was 0.10% at a velocity of 4.4 m/s. Subsequent modification to the electromagnetic shielding of the data acquisition system eliminated signal noise, and the turbulence level is now 0.07% at a velocity of 3.3 m/s. It is possible to continuously vary the working section velocity from 1 m/s to 50 m/s. The tunnel is also highly uniform with a coefficient of variation of 0.0027.

The velocity measurement is performed with a TSI *IFA-100* constant-temperature anemometer, and a TSI *1210-T1.5* hot-wire probe (Diameter of probe is 4 μm). The signal is processed through a Rockland *852* band-pass filter, and acquired with a Boston Technology *PC-30DS* 12-bit ADC card mounted on a IBM-PC.

An X-Y motorised linear traversing mechanism has been constructed to traverse a hot-wire across the near-wake of a circular cylinder. The mounting-stage is moved by a THK *MTF-0601* ball-screw mechanism with a lead of 1 mm, and controlled by a Superior Electric *Slo-Syn* stepping motor which steps through 200 steps/revolution. The movement of the traverser is 5 microns with an accuracy of ±0.13 microns. The traverser enables the characterisation of the separated shear layer in the near-wake of a circular cylinder to be performed in a similar manner done on the shear layer of a jet by Freymuth (1966). Velocity profiles of the near-wake are taken at several downstream stations.

Two cylinders were used in the investigation. They are 244 mm long with diameters of 12.7 and 6.4 mm. These were made of perspex. The origin of coordinates was taken at the centre of the cylinders midway along their length. The *x*-axis was measured in the direction of the flow, the *y*-axis was perpendicular to the flow, and the *z*-axis coincided with the cylinder axis.

RESULTS AND DISCUSSION

The experiment was conducted over a freestream velocity range of 4.2–5.2 m/s which covers a *Re* range of approximately 1810–3590. Table 1 tabulates some pertinent information on the experiments conducted. The *x*- and *y*-station separation refers to the distance between sampling stations in the *x*- and *y*-directions. The traverser motor steps through up to 60 steps between cross-stream stations. If required, the resolution can be increased by reducing the number of these steps. The results are as presented below.

Table 1. Experiment set-up

Cylinder diameter (mm)	*Re*	*x*-station separation (*D*)	*y*-station separation (*D*)
6.4	1810	0.078	0.0047
6.4	2020	0.078	0.0047
6.4	2220	0.078	0.0047
12.7	3590	0.079	0.0047

Mean velocity profiles

The mean velocity profiles are shown in Fig. 2. These are taken at the *x*-station $x/D = 0.656$ for $D = 6.4$ mm and $x/D = 0.657$ for $D = 12.7$ mm. The *y*-coordinates were non-dimensionalised using $(y - y_{0.5})/y_{0.5}$, where $y_{0.5}$ is the co-ordinate where $U(y) = 0.5U_{max}$.

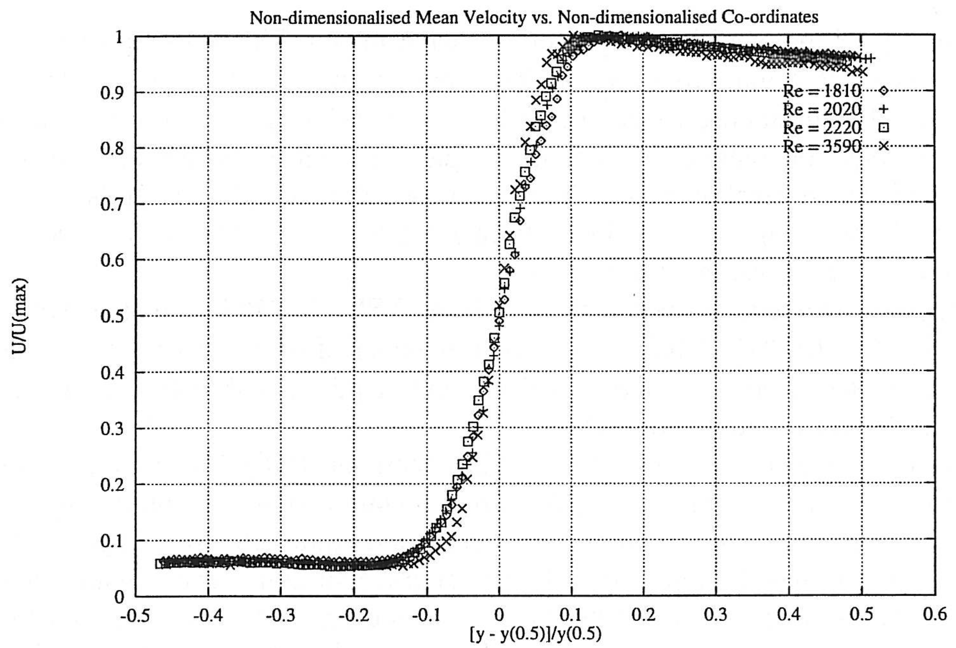


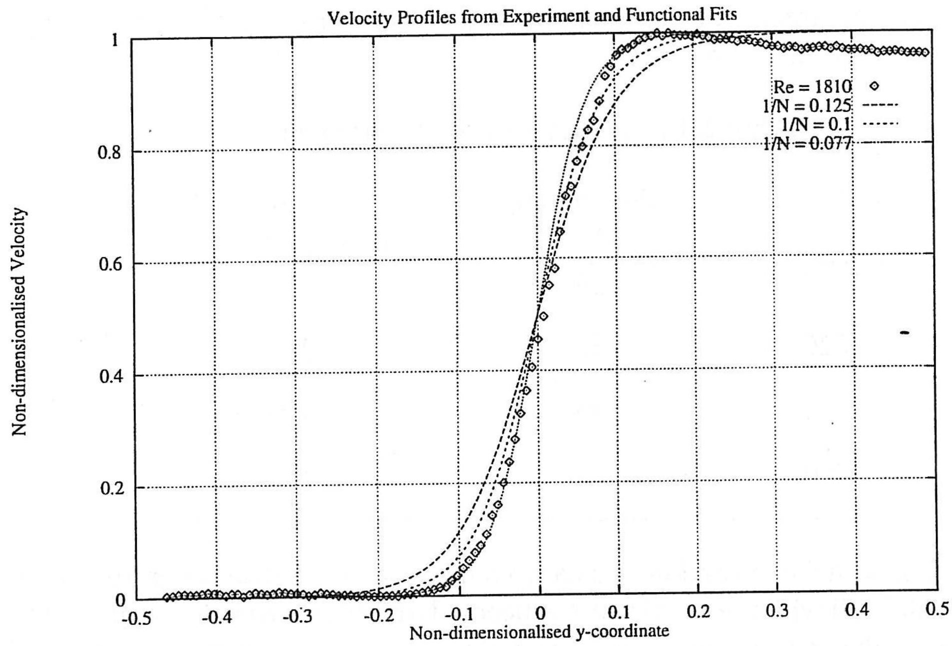
Figure 2. Mean velocity profiles. $x/D \approx 0.66$.

Function curves of the form

$$\frac{U}{U_{max}} = 1 - \left[1 + \sinh^{2N}\left[(y+1)\sinh^{-1}1\right]\right]^{-1}$$

are fitted to the profiles from experiment to perform stability analysis. This single-parameter family of function was derived from that used by Monkewitz & Nguyen (1987) for their analysis. The parameter $1/N$ provides a measure of the vorticity-thickness. Monkewitz & Nguyen (1987) found that at the transition $1/N = 0.08$, the wake characteristic changes from an absolutely unstable nature to a convective one.

Function fits for the *Re* range of the present study is shown in Fig. 3. It was found that the near-wake of the circular cylinder is convectively unstable up to approximately $x/D = 0.66$ where the wake has already 'cleared' the cylinder. According to Monkewitz & Nguyen (1987), this transition point acts as a reflector and the cylinder as a broadband reflector for a self-sustaining feed-back loop contained within the initially convective instability region. It was also shown that such a loop is feasible even though the spatial amplification rates revealed that the attenuation of the signal larger than the gain. This is because the difference between attenuation and gain is small, and moderate gains at the body is enough to sustain the loop.



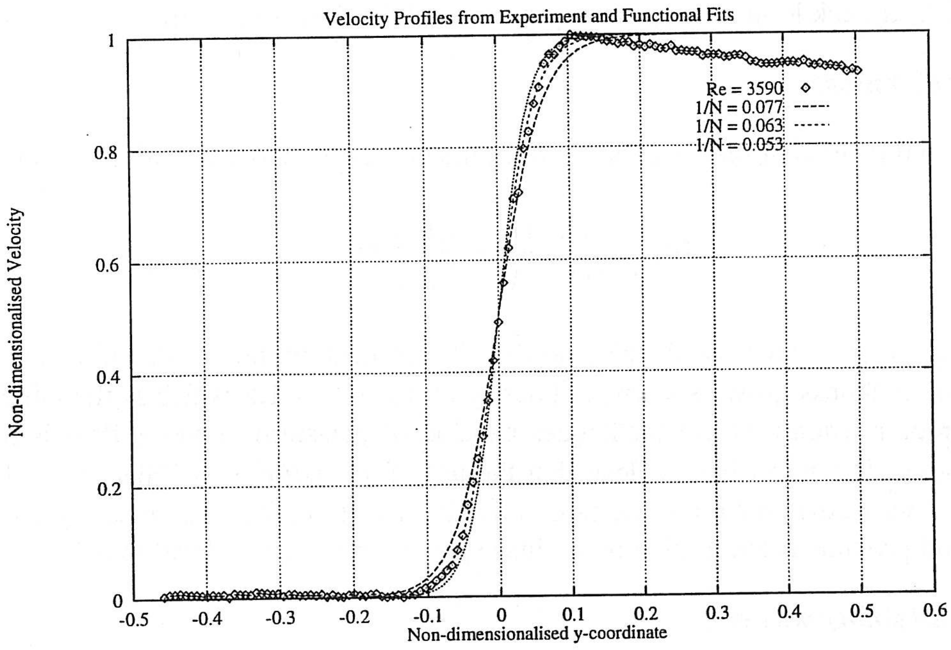


Figure 3. Velocity profiles from experimental data and the single-parameter family of functions.

Assuming that there is another transition point further downstream where the wake reverts to a convectively unstable nature, the wake of a circular cylinder can therefore be classified as type "Absolute Free" (Monkewitz & Nguyen, 1987) where a region of absolute instability is bounded by two transition points with no solid boundaries in their streamwise vicinity.

It is conceivable that the upstream transition point could also function as a reflector for instability waves with the second reflector being the downstream transition point. For the global resonance to be the branch-point frequency selected at both ends of this must be similar. Presently, any downstream transition point branch-point frequency could not be determined due to the non-availability of suitable experimental velocity. The frequency predicted at the upstream transition point is $\omega^0 = 1.68$ (Monkewitz & Nguyen, 1987).

Table 2 presents the frequency data from the experiment for comparison with the predicted branch-point frequency, $\omega^0 = 1.68$.

Table 2. Frequency data from experiment

Re	Measured f_K (Hz)	ω ($4\pi f_K y_{0.5}/U_{max}$)
1810	129.2	1.56
2020	152.6	1.63
2220	170.3	1.69
3590	58.7	1.41

As can be seen, with the exception of data from the $D = 12.7$ mm cylinder, the experimental data matches quite well with the predicted frequency. Further experiment is expected to clarify the discrepancy of the larger cylinder data, and also to investigate further the feasibility of a self-sustained feed-back loop between the upstream and the downstream transition point.

Momentum thickness

The momentum thickness of the shear layers from the circular cylinder is evaluated using

$$\theta = \int_{-\infty}^{\infty} \frac{U(y)}{U_{max}}\left(1 - \frac{U(y)}{U_{max}}\right) dy$$

Figure 4 shows the streamwise development of the momentum thickness, and it is clear that the momentum thickness grows somewhat linearly up till $x/D \approx 1.5$ which is the edge of the recirculating region. Measurement techniques capable of measuring reverse flow is required for further studies. It is nevertheless clear, that the use of $\theta \approx D/Re^{0.5}$ by Filler *et al.* (1991) is inaccurate. Further studies with the insertion of a splitter plate in the wake to delay the vortex formation could provide a clearer picture on this aspect of shear layer development.

Shear layer instability waves

The most interesting result from the present study up to this stage is the shear layer instability waves, or rather the lack of it. Spectral analysis of the linearised hot-wire signals at every point through the shear layer at all the *Re* investigated showed no peak that can be associated with the high-frequency shear layer instability waves. Figure 6 in Unal & Rockwell (1988) showed that at certain sections across the shear layer, signals from the shear layer instability waves appeared to be stronger than that from the Kármán vortex.

To speed up further investigations, more spectral analyses were performed on unlinearised signals taken from the wakes at *Re* between 1000 and 13000 (a cylinder of $D = 25.4$ mm was also used). All these revealed no detectable signs of the shear layer instability waves.

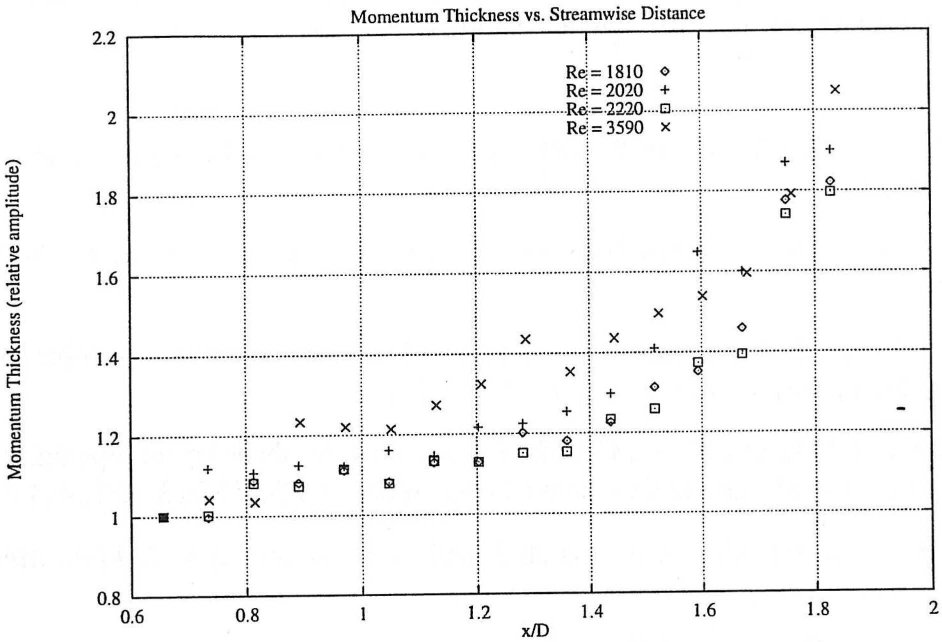


Figure 4. Momentum thickness as a function of downstream distance.

It is speculated here that the shear layer instabilities are triggered by freestream turbulence. From previous studies, it appears that the lower the free-stream turbulence level, the higher the *Re* at which these waves first appear (cf. Unal & Rockwell, 1988 - turbulence level = 0.06%, *Re* = 1800; Wei & Smith, 1986 - turbulence level = 0.3%, *Re* = 1100). Filler *et al.* (1991) also showed that shear layer waves can be excited by applied perturbation at lower *Re* (shear layer response was obtained at *Re* as low as 575 with a perturbation level of 2.2%).

Experiments planned include investigating the effects of different turbulence levels as well as acoustic perturbations on the shear layer response.

CONCLUSION

Velocity profiles for the wake of circular cylinders were measured for *Re* between 1810 and 3590. Functional fits to these profiles showed that the near-wake of circular cylinders is convectively unstable up to 0.66 where is exhibits absolute instability. Measurement of the Kármán vortex frequency indicated that there is good agreement with the branch-point frequency of the transition point predicted by stability analysis. Pending further studies, the mechanism by which the global resonance loop is sustained remains unclear.

Measurement of the momentum thickness indicated nearly-linear growth up to $x/D = 1.5$ which is the edge of the recirculation zone.

The shear layer instability waves were not detected in the course of this study. It was speculated based on previous reports that this is due to the low-level of freestream turbulence in the wind tunnel used. Future efforts include investigating the effects of freestream turbulence on the shear layer response, and how this in turn affects the Kármán vortex formation.

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REFERENCE

Betchov, R. & Criminale, W. O. 1966 Spatial instability of the inviscid jet and wake *Phys. Fluids* **9**, 359–362.

Bloor, M. S. 1964 Transition to turbulence in the wake of a circular cylinder. *J. Fluid Mech.* **4**, 313–340.

Chomaz, J. M., Huerre, P. & Redekopp, L. G. 1988 Bifurcations to local and global modes in spatially developing flows. *Phys. Rev. Lett.* **60**, 25–28.

Filler, J. R., Marston, P. L. & Mih, W. C. 1991 Response of the shear layers separating from a circular cylinder to small-amplitude rotational oscillations. *J. Fluid Mech.* **231**, 481–99.

Freymuth, P. 1966 On transition in a separated laminar boundary layer. *J. Fluid Mech.* **25**, 683–704.

Gorman, J., Soria, J., Thompson, M. C., Welsh, M. C. & Parker, R. 1992 Flow around a lifting aerofoil: absolute instabilities and vortex shedding. *Proc. 11th Australasian Fluid Mechanics Conference*, 14–18 December 1992, Hobart, Australia. pp. 985–988.

Ho, C.-M. & Huerre, P. 1984 Perturbed free shear layers. *Ann. Rev. Fluid Mech.* **16**, 365-424.

Huerre, P. & Monkewitz, P. A. 1985 Absolute and convective instabilities in free shear layers. *J. Fluid Mech.* **199**, 55–88.

Koch, W. 1985 Local instability characteristics and frequency determination of self-excited wake flows. *J. Sound Vib.* **99**, 53–83.

Kourta, A., Boisson, H. C., Chassaing, P., Ha Minh, H. 1987 Nonlinear interaction and the transition to turbulence in the wake of a circular cylinder. *J. Fluid Mech.* **181**, 141–161.

Michalke, A. 1965 On spatially growing disturbance in an inviscid shear layer. *J. Fluid Mech.* **23**, 521-544.

Monkewitz, P. A. 1988 The absolute and convective nature of instability in two-dimensional wakes at low Reynolds numbers. *Phys. Fluids* **31**, 999–1006.

Monkewitz, P. A. & Nguyen L. N. 1987 Absolute instability in the near-wake of two-dimensional bluff bodies. *J. Fluids Struct.* **1**, 165–184.

Unal, M. F. & Rockwell, D. 1988 On vortex formation from a cylinder. Part 1. The initial instability. *J. Fluid Mech.* **190**, 491-512.

Wei, T. & Smith, C. R. 1986 Secondary vortices in the wake of circular cylinders. *J. Fluid Mech.* **169**, 513–533.

Wu, J. 1992 An experimental investigation of the separating shear layer behind a circular cylinder. Ph. D. *Proj. Rep.*, Monash University.