SHEAR LAYER VORTICES AND LONGITUDINAL VORTICES IN THE WAKE OF A CIRCULAR CYLINDER

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ABSTRACT

In this paper, results and discussions will be given on the small-scale shear layer vortices developing in the free shear layers located between the separation point and the first shed Strouhal vortex of a circular cylinder and their relationships to the longitudinal vortices developing in the braid region of the Strouhal vortices in the wake. Evidence based on flow visualisation will be given to show that the longitudinal vortices develop independently at low Reynolds numbers. At higher Reynolds numbers, it will be shown that the spanwise length scale of the longitudinal vortices in the separated shear layers and the longitudinal vortices in the Strouhal vortices are comparable near $Re \approx 900$. Based on this evidence it is suggested that coupling between the two vortices is possible at this Reynolds number.

INTRODUCTION

The shedding of Strouhal vortices from a circular cylinder results from the interaction of the two free shear layers separated from the cylinder. It is also known that small-scale vortices due to the Kelvin-Helmholtz instability develop in the separated free shear layers prior to the formation of Strouhal vortices. Bloor (1964) appears to have been the first to observe these secondary vortices. She noted their similarity to the Tollmien–Schlichting waves seen in boundary layers and suggested their possible importance as a route in the transition to turbulence. Wei and Smith (1986), using a flow visualisation technique and hot-wire anemometry, examined this phenomenon in the Reynolds number range 1200-11000. They established a 0.87-power law relationship between the shear layer instability frequency and the Reynolds number. Kourta *et al.* (1987), using hot-film anemometry and flow visualisation, studied the problem in the Reynolds number range 2000-60000. Their results showed that shear layer instability waves can be clearly identified in the power spectrum of velocity measured just downstream of the separation point.

Another interesting feature of the wake behind a circular cylinder is the development of longitudinal vortices (Williamson 1988; Wu *et al.* in press). They are found to be counter-rotating shear-aligned vortex structures superimposed on the Strouhal vortices in the braid region. It is interesting to investigate possible relationships between the two vortex systems, i.e. the shear layer vortices and the longitudinal vortices. Wei and Smith (1986) proposed that the distortion of the shear layer vortices and subsequent stretching by the Strouhal vortices are responsible for the onset of the counter-rotating longitudinal vortices.

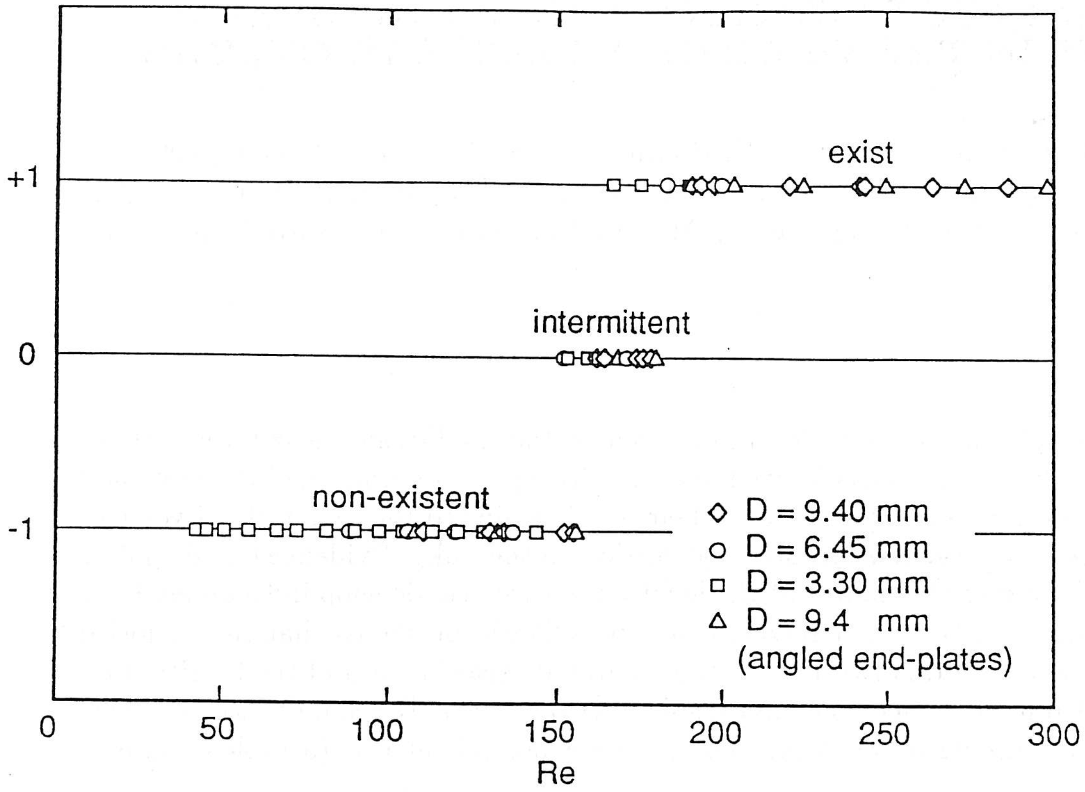

Figure 1: Onset of the longitudinal vortices: effect of Reynolds number. The existence of the longitudinal vortices: non-existent, intermittent and existent, and represented by -1, 0, +1 respectively.

In this paper, results based on flow visualisation will be presented and the relationship between the two vortex systems will be discussed. The spanwise length scale presented in this paper enables an analysis of possible interaction between the two vortex systems.

CRITICAL REYNOLDS NUMBER

Onset of longitudinal vortices

The critical Reynolds number Re_{cr} for the onset of the longitudinal vortices was investigated using flow visualisation in a water tunnel described in Wu *et al* (in press). Observations focused on the near wake region extending up to $10D$ (D is the cylinder diameter) downstream of the cylinder. As the velocity was low in the Reynolds number range under investigation, it was possible to observe directly the onset of longitudinal vortices in the wake when the freestream velocity was gradually raised. The test was also conducted when the Reynolds number was reduced from above the critical Reynolds number, until the longitudinal vortices disappeared completely. The test was repeated several times during a week, and one month after, in order to establish confidence in the results. Three different cylinder diameters, i.e. D = 3.3 mm, 6.4 mm, 9.4 mm were used during the tests.

The results are graphed in Fig. 1. The status of the longitudinal vortices is divided into three levels: non-existent, intermittent and existent, and digitally represented by -1, 0, +1 respectively. The intermittent status describes a situation where the vortices were found to exist intermittently. The longitudinal vortices were not observed below the intermittent region of $Re \approx 150$–175, and found to exist over the full cylinder span above this region.

Onset of shear layer vortices

Flow visualisation tests were conducted in the same water tunnel to examine the vortices developing in the separated shear layers from a circular. In one test, a hydrogen bubble wire (upstream wire) was placed normal to the axis of the cylinder (D=25.1mm). In another test, a hydrogen bubble wire was flush mounted on the surface of the cylinder 60 degrees from the upstream stagnation point and parallel to the cylinder axis. The results are summarised in Table 1.

Table 1: Flow visualisation observation of the onset of small-scale vortices in the separated free shear layers of a circular cylinder, D=25.1 mm.

Observation of shear layer vortices		
Re	upstream wire	flush-mounted wire
500	non-existent	non-existent
1000	non-existent	existent
2000	dubious	existent
3000	existent	existent

The results presented in the table suggest that the minimum Reynolds number for shear layer vortices to appear is approximately 2000 $\sim$ 3000 when tested using the upstream wire method and approximately 1000 when using the flush-mounted wire method. This discrepancy was caused by the flush-mount technique. It was recognised during the experiments that minute hydrogen bubbles accumulated on the wire flush-mounted on the body surface, which increased surface roughness. The surface roughness disturbed the boundary layer formed along the body causing the development of the shear layer vortices at lower Reynolds numbers; this is expected since the free shear layer is susceptible to external disturbance (Wu *et al.* 1992; Sheridan *et al.* 1992).

It should be pointed out that Unal and Rockwell (1988) found that the critical Reynolds number $Re \approx 1900$ and Bloor (1964) found $Re \approx 1300$ at which the shear layer vortices were seen to develop. The discrepancy is quite large and perhaps could be attributed to a difference in the background disturbance conditions.

Nevertheless, it can be concluded that the critical Reynolds number for the onset of shear layer vortices is approximately 1000 $\sim$ 3000. The precise value is background disturbance dependent, a high turbulence background may cause a reduction in the critical Reynolds number. This range is significantly higher than the critical Reynolds number for the onset of longitudinal vortices embedded in the Strouhal vortices.

Spanwise Length Scale Of The Two Vortices

When the Reynolds number is above 2000 $\sim$ 3000, small-scale vortices will develop in the two free shear layers before merging into the Strouhal vortices. It has been established that streamwise vortices are found superimposed on the spanwise vortices that develop in plane mixing layers (Bernal and Roshko 1986). It is clear that streamwise vortices should develop and be superimposed on the vortices in the separated shear layers of a cylinder, prior to the formation of the Strouhal vortices at $Re > 2000 \sim 3000$.

In order to estimate the spanwise length scale of the streamwise vortices known to be superimposed on the spanwise vortices in the separated shear layers, the frequency of the shear layer vortices was measured using an acoustic perturbation method. This method used two loud-speakers connected in anti-phase positioned at opposite sides of the test section to produce a sinusoidal acoustic standing wave, and a hot-wire anemometer to measure velocity fluctuations (see Sheridan *et al.* 1992). The perturbation method was employed to avoid inaccuracy which could be caused by wind tunnel noise as the shear layer is extremely sensitive to disturbance. It was shown that our results are in good agreement with the following correlation suggested by Kourta *et al.* (1987):

$$\frac{f_{BG}}{f_{KM}} = 0.095 Re^{0.5} \tag{1}$$

where the subscript BG is used in honour of Bloor and Gerrard, who were among the first to report the shear layer vortices, the subscript KM denotes the *Kármán* vortices, and $f_{KM} \approx 0.2U_0/D$. The convective velocity of the shear layer vortices was assumed to be $U_c = (U_1 + U_2)/2$, where U_1, U_2 are flow velocities on the high and low speed sides of the shear layer; based on hot-wire measurements conducted near the free shear layers (Wu *et al.* 1992), $U_2 \approx 0.0$, $U_1 \approx 1.3U_0$, thus we get $U_c = 0.65U_0$. The streamwise wavelength of the vortices is:

$$\lambda_{BG} = \frac{U_c}{f_{BG}} = \frac{0.65U_0}{0.095Re^{0.5} f_{KM}} = \frac{34D}{Re^{0.5}} \tag{2}$$

or:

$$\frac{\lambda_{BG}}{D} = \frac{34}{Re^{0.5}} \tag{3}$$

On the other hand, we note that the streamwise vortices which develop on top of the spanwise vortices in a mixing layer have a wavelength ratio of 0.67 (Bernal and Roshko 1986). If we assume that the separating shear layers have the same wavelength ratio between the streamwise and spanwise vortices, then:

$$\lambda_{BG3} = 0.67\lambda_{BG} \tag{4}$$

Here λ_{BG3} is the spanwise wavelength of the three-dimensional vortices superimposed on the shear layer vortices. Substituting the expression for λ_{BG} into (4) we get:

$$\frac{\lambda_{BG3}}{D} = \frac{23}{Re^{0.5}} \tag{5}$$

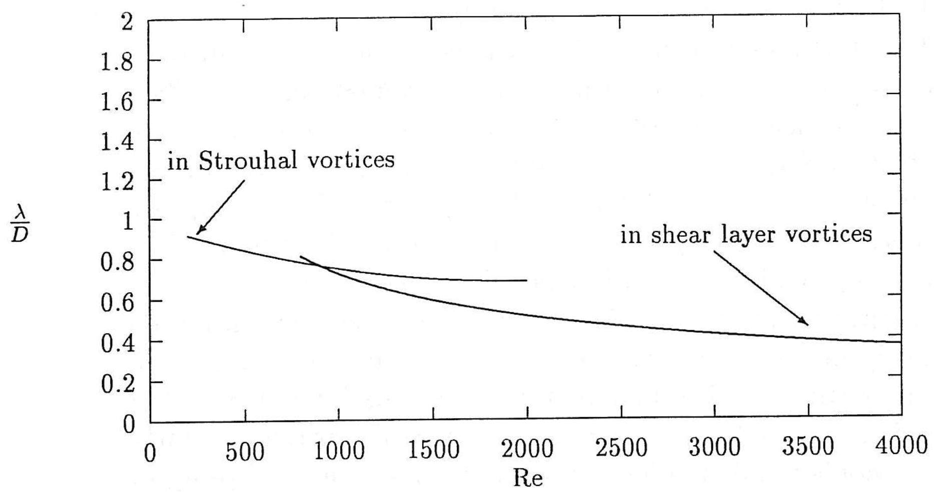

Figure 2: A comparison of spanwise wavelength: streamwise vortices in the separated shear layers and in the Strouhal vortices.

The spanwise wavelength of longitudinal vortices superimposed on Strouhal vortices were also measured using a flow visualisation method for Reynolds numbers ranging from 250 to 1800; detailed results have been presented in Wu *et al.* (in press). The two spanwise wavelength curves, representing those streamwise vortices in the separated shear layer and those in the Strouhal vortices are graphed in Fig. 2. It is seen that for $Re > 2000$, the two structures have quite different length scales, suggesting they might be effectively decoupled. As the Reynolds number is increased, the size of the shear layer streamwise vortices (longitudinal vortices) reduces faster than those superimposed on the Strouhal vortices.

However, when the Reynolds number is reduced to around $Re \approx 900$, the two curves intersect, indicating that the two structures have comparable spanwise length scales. Coupling might therefore be possible in this Reynolds number region, if the shear layer vortices exist (or at least are excited by some form of disturbance).

DISCUSSION AND CONCLUSION

The development of small-scale free shear layer vortices and the development of the counter-rotating longitudinal vortices in the braids joining consecutive Strouhal vortices are the main features of near wake vortex dynamics. The results presented in this paper show that the onset of the two types of vortices is Reynolds number dependent and the critical Reynolds numbers for the two vortices to first appear are significantly different. The results indicate that the longitudinal vortices develop independently from the shear layer vortices at low Reynolds numbers, as the former has a much lower critical Reynolds number than the latter. At a higher Reynolds number, however, through an analysis of spanwise length scale, we suggest that the coupling between the two vortices might be possible as the spanwise length of the two vortices are comparable.

Wei and Smith (1986), in their flow visualisation investigations, found that small-scale vortices develop in the free shear layers prior to the formation of Strouhal vortices. They showed that cellular patterns in the near wake region (where shear layer vortices are located) are connected to the streamwise vortices (longitudinal vortices) in downstream locations, leading them to speculate that some form of feedback might exist. It appears that, based on the evidence presented in this paper, coupling between the two vortices existed in their experiment due to probably a match in spanwise wavelength.

In conclusion, some experimental results and analyses have been presented in an attempt to illustrate the relationship between the two vortices, i.e. small-scale vortices in the free shear layer and counter-rotating longitudinal vortices in braids joining Strouhal vortices. It is shown that the critical Reynolds numbers of the two vortices are significantly different and coupling at low Reynolds numbers ($Re \approx 175$) is not possible. Coupling at higher Reynolds numbers is however possible, as the spanwise length scale of the two vortices could then be comparable.

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