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On Axisymmetric Bluff Body Wakes: Three-Dimensional Wake Structures and Transition Criticality of the Torus

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Three bluff ring aspect ratios are studied numerically to elucidate the various three-dimensional transition modes previously predicted in the wakes of bluff rings by linear stability analysis. The transitions are modeled with respect to the Landau equation, and the criticality of the various transitions is determined from the coefficients of the Landau equation.
The wake flow fields are simulated using a spectral-element scheme, incorporating a Fourier expansion of the two-dimensional grids in the spanwise direction to evolve the three-dimensional wake flow.
Linear Floquet stability analysis performed previously on the wakes of bluff rings has predicted numerous three-dimensional transitions at various aspect ratios. Bluff rings with smaller aspect ratios (those approaching the sphere geometry) are predicted to undergo a regular three-dimensional transition (ie. steady to steady flow), followed by a Hopf bifurcation to an unsteady wake with increasing Reynolds number. Three transition modes have been identified in this aspect ratio range, referred to as Modes I, II and III respective to increasing aspect ratio.
Plots visualizing the aforementioned transition modes are provided, and the transitions are modeled with the Landau equation, allowing the criticality of the various modes to be determined.

1. Introduction

The various transitions in the wake of bluff rings have been studied over the past decade (Leweke & Provansal, 1995; Sheard et al 2001, 2002) with a rich diversity of wake states being determined. The bluff ring geometry is of interest as a fundamental bluff body due to its inherent characteristic whereby the adjustment of a single geometric parameter (aspect ratio, *Ar*) allows myriad geometries to be represented. By varying the single geometric parameter *Ar*, a uniform axisymmetric body is described varying from a sphere at $Ar = 0$, to a straight cylinder at the limit $Ar \rightarrow \infty$. Figure 1 shows a schematic diagram of the bluff ring system.

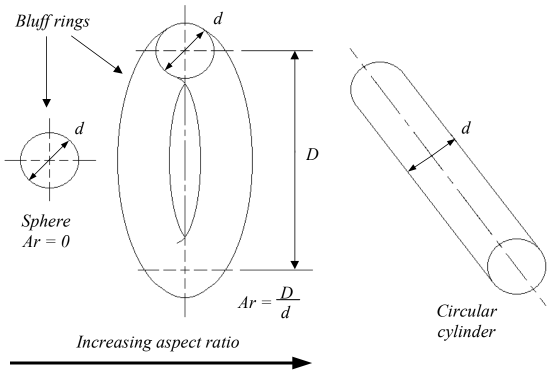


FIGURE 1. Schematic diagram of the bluff ring system.

This paper is concerned with the wake transitions of axisymmetric bodies (ie. the sphere), hence the aspect ratio range $0 \le Ar \le 4$ is considered. It has been predicted from linear stability analysis (Sheard et al, 2001, 2002) that three distinct three-dimensional transition modes exist over this range of aspect ratios, as shown in figure 2.

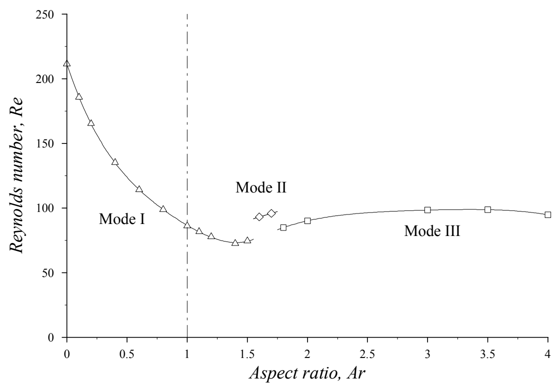


FIGURE 2. Plot showing predicted critical transition Reynolds number profiles for dominant three-dimensional stability modes in the wake of rings with aspect ratios $0 \le Ar \le 4$. The mode I and mode III transitions are regular transitions, and the mode II transition is a Hopf transition. The dotted line shows the aspect ratio at which the hole through the center of the ring emerges.

Aspect ratios Ar = 0.6, 1.6 and 2.0 are chosen to isolate each of the three three-dimensional transition modes identified in previous work (mode I, II and III transitions respectively). Three-dimensional numerical simulations employing a spectral-element method and an azimuthal Fourier expansion (see Thompson et al 1996, 2001 for a description of the numerical method, and Sheard et al 2001, 2002 for grid resolution studies) are used to simulate the three-dimensional wakes arising from these transitions. The Reynolds number incorporates the diameter ***d*** for a length scale.

The criticality of a transition describes the behaviour of the unstable mode. Transitions that occur through a supercritical bifurcation such as the regular and Hopf transitions of the sphere wake (Thompson et al 2001) are not hysteretic. A subcritical transition, however, displays hysteretic properties, with the stable and unstable flow states able to exist in the vicinity of the critical Reynolds number for the transition. The Landau equation will be used to determine the criticality of the three-dimensional modes of the bluff rings. In the Landau equation the growth of the unstable mode is described by a differential equation for the complex amplitude (A) of the unstable mode that varies in time (t):

$$\frac{dA}{dt} \approx \sigma A - lA^3$$

Criticality is determined from the sign of the l-term. A positive l-term allows the amplitude to saturate, and hence describes a supercritical transition. A negative l-value requires a higher-order amplitude term to be added to the right-hand side of the equation to allow saturation, and hence describes a subcritical transition.

The "point method" (described in Thompson et al, 2001) is applied in the present study. It relies on the assumption that at any point in the near wake the transition behaviour will be representative of the global behaviour of the flow field. For the stability of Hopf bifurcations, phase information is not considered, and the saturation of the envelope of oscillation of the azimuthal mode is treated as per the regular transitions.

4. Results.I. Visualising the Small Aspect Ratio 3D Transition Mode Wakes

The three-dimensional wakes following the mode I, II and III transitions are represented by streamwise vorticity isosurface plots in figure 3.

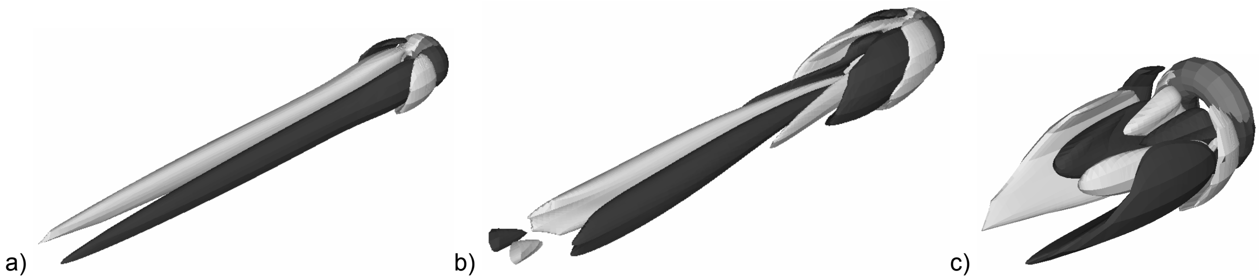


FIGURE 3. Isosurface plots of streamwise vorticity indicating three-dimensional wake structures for bluff ring wakes following the mode I, II and III transitions. The mode I wake for the Ar = 0.6 ring at Re = 130 is shown in (a), the mode II wake for the Ar = 1.6 ring at Re = 100 is shown in (b), and the mode III wake for the Ar = 2.0 ring at Re = 100 is shown in (c). Light grey isosurfaces represent negative streamwise vorticity, and dark grey isosurfaces represent positive streamwise vorticity. The bluff ring is located at the upper right corner of each frame, and flow is from the top right to the bottom left in each case.

It is clear from the isosurfaces plots in figure 3 that a plane of symmetry exists in the wakes through the centre of the ring for all modes. This symmetry has been observed for the sphere (Tomboulides & Orszag, 2000; Johnson & Patel, 1999), which is another example of the mode I transition. Note that an m = 1 azimuthal symmetry maintained for each mode, as predicted by linear stability analysis (Sheard et al, 2002). In figure 3a, wings of streamwise vorticity immediately behind the ring are observed, as is a pair of vorticity tails stretching far downstream. These structures are indicative of the classic "double threaded wake" observed in the wake of the sphere (Tomboulides & Orszag, 2000; Thompson et al, 2001) following the first three-dimensional transition.

The near wake following the mode II transition (figure 3b) is similar to the mode I near wake region, with wings of streamwise vorticity of opposing sign wrapped around longer tails of streamwise vorticity extending downstream. These tails are not steady, but are shed downstream. The mode II transition is a Hopf transition from a steady axisymmetric flow to a three-dimensional unsteady flow. This verifies the prediction of Sheard et al (2001, 2002), that the dominant Floquet mode has an imaginary component, resulting in a Hopf transition to an unsteady three-dimensional wake.

The wake of the ring following the mode III transition is shown in figure 3c. This mode is a regular (steady-steady) transition to three-dimensionality, as is mode I. Structurally the wake is vastly different to the mode I transition wake. There is an absence of three-dimensional vortical structures along the axis, and instead bands of vorticity are located directly downstream of the ring body. These structures indicate that the transition involves a loss of azimuthal stability of the recirculating rings behind the body. This agrees with the previous stability analysis of Sheard et al (2002).

5. Results.II. Landau Modelling of the Mode I & III 3D Transitions

The non-linear behaviour of the regular mode I and mode III transitions is studied by determining the Landau coefficients for an azimuthal velocity transient at a point directly downstream of the cross-section of each ring. The criticality of each mode will be determined, and where applicable comparisons will be drawn to previous work on sphere stability (Thompson et al, 2001a) and bluff rings (Sheard et al, 2002). The mode II Hopf transition, and the Hopf transitions following the Mode I and Mode III regular transitions are studied later.

5.1. *The Mode I Transition*

The mode I transition has been found to be supercritical for the *Ar* = 0.6 ring, in agreement with Thompson et al (2001a) on the sphere wake transition (another mode I transition). The plots in figure 4 illustrate the supercritical behaviour.

The critical Reynolds number for the mode I transition was determined from the growth rate, σ, from the Landau model. The resulting critical Reynolds number matched those predicted by the linear Floquet stability analysis of Sheard et al (2002) to within 0.5%, with Re_c = 114 for the *Ar* = 0.6 ring being found.

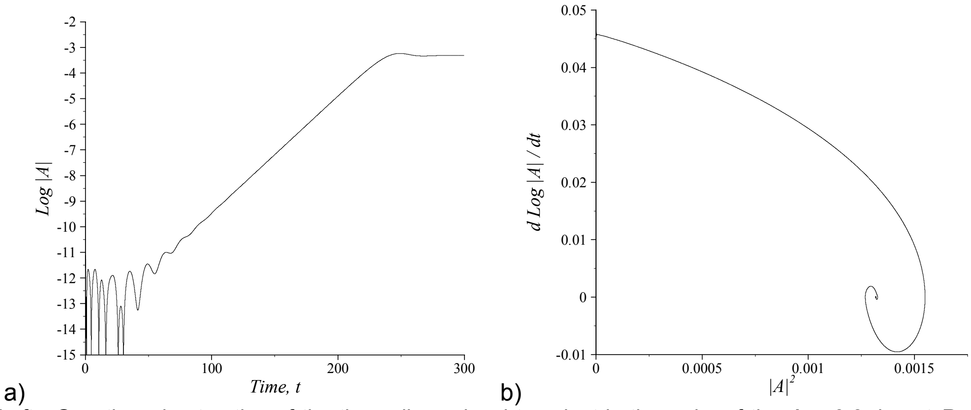


FIGURE 4. (a) Left: Growth and saturation of the three-dimensional transient in the wake of the *Ar* = 0.6 ring at *Re* = 130. (b) Right: Amplitude derivative versus amplitude squared plot. Y-axis intercept gives growth rate, σ, and gradient close to y-axis gives saturation term, *l*. The negative slope and linear behaviour near the y-axis indicate that the transition is supercritical.

5.2. *The Mode III Transition*

The Mode III transition is a regular transition (Sheard et al (2002). Landau modelling of the non-linear behaviour of the transition shows it to be subcritical. The behaviour of this transition for a ring with *Ar* = 2 at *Re* = 98 is shown in figure 5. Figure 5b shows the subcritical behaviour, with a distinct non-linearity and positive gradient at the y-intercept. Thus higher order terms are required in the Landau equation to adequately model the growth of the transition to saturation.

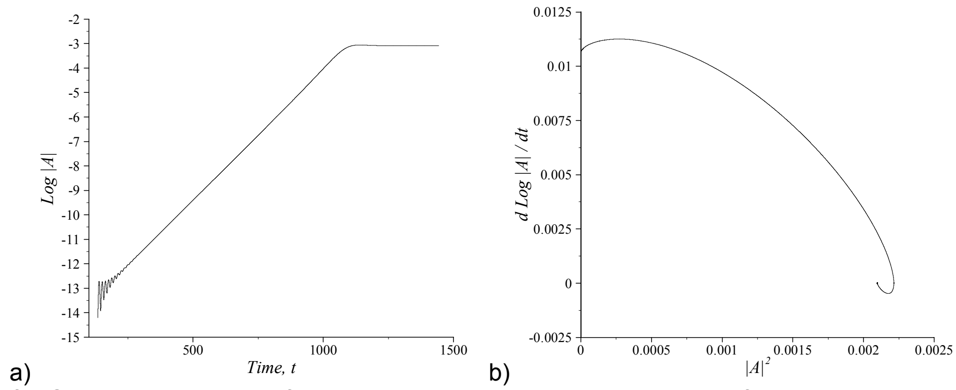
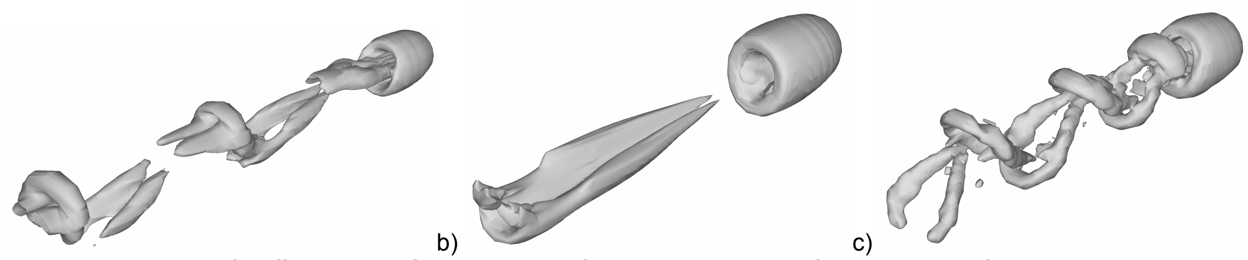


FIGURE 5. (a) Left: Growth and saturation of the three-dimensional transient in the wake of the *Ar* = 2 ring at *Re* = 98. (b) Right: Amplitude derivative versus amplitude squared plot. Y-axis intercept gives growth rate, σ, and gradient close to y-axis gives saturation term, *l*. The positive gradient and non-linear behaviour near the y-axis indicate that the transition is subcritical.

6. Results. III. Visualising the Hopf Transition Modes of Small Aspect Ratio Ring Wakes

This section provides visualisations of the wakes of small aspect ratio rings at Reynolds numbers above the critical Reynolds numbers for the Hopf transition. Hussain field isosurfaces are utilised to capture the three-dimensional vortical structures observed in the wakes.

The regular mode I and mode III transitions are followed by a secondary Hopf transition to unsteady flow at higher Reynolds numbers. For the sphere wake this transition occurs at Re = 272 (Thompson et al, 2001a; Johnson & Patel, 1999), and the resulting wake is characterised by a plane of symmetry along the axis, and an azimuthal symmetry *m* = 1. The wake structure consists of hairpin shaped vortex loops shedding alternately from opposite sides of the axis. The plots in figure 6 show similar “hairpin” structures in the wakes of both the *Ar* = 0.6 and *Ar* = 2.0 rings subsequent to the mode I and mode III transitions. An *m* = 1 azimuthal symmetry is observed for each wakes presented here, and a plane of symmetry exists along the axis in each case.



a) b) c)

FIGURE 6. Visualisation of bluff ring wakes following the Hopf transition. The wake following the Hopf transition secondary to the mode I transition is shown in (a) for the Ar = 0.6 ring at Re = 160. The mode II Hopf transition wake is shown in (b) for the Ar = 1.6 ring at Re = 100. The wake following the Hopf transition in the mode III regime is shown in (c) for the Ar = 2.0 ring at Re = 150. Isosurfaces represent Hussain field to elucidate vortical structures in the wake. Note the plane of symmetry through centre of each ring, and similar vortical structure of the wakes. Flow is from top right to bottom left in each frame.

The mode II wake (figure 6b) differs from the other wakes in that the Strouhal frequency of the oscillation was not periodic, and the average Strouhal frequency obtained was also far lower (about 30% of the Strouhal frequencies of the wakes in the mode I and mode III transition regime. The Hussain field isosurfaces plot of the mode II wake has been obtained from the same velocity field as the streamwise vorticity plot in figure 6b. The mode II wake doesn't exhibit a hairpin wake. Instead, long vortical folds are cast into the wake from one side of the axis.

7. Results. IV. Landau Modelling of the Hopf Transitions for Small Aspect Ratio Rings

The non-linear behaviour of the Hopf transitions of bluff rings with aspect ratios in the range $0 \le Ar \le 4$ is examined by application of the Landau equation, and the results are summarised here.

The Hopf transition following the mode I transition for the Ar = 0.6 ring is a supercritical Hopf transition. The mode II transition is found to occur through a subcritical bifurcation of the steady axisymmetric wake, to a three-dimensional unsteady wake. The criticality of the Hopf transition that occurs in the ring wake following the mode III transition is supercritical. Interestingly, the Hopf transition following the mode III transition is supercritical, as is the Hopf transition following the mode I transition, despite the regular mode III transition being subcritical. This supports the observations of the wake structures, indicating the transitions to be similar, as both wakes consist of hairpin vortices shedding downstream.

10. Conclusions

Three-dimensional isosurface plots of streamwise vorticity and Hussain fields have enabled wake structures characterizing the regular and Hopf transitions of bluff rings with $Ar < 4$ to be identified.
By determining the Landau coefficients for the non-linear behaviour of azimuthal velocity transients in the wake of these bluff rings, the criticality of the various transitions was determined.
The regular mode I transition was found to be supercritical, as was the subsequent Hopf transition over the same aspect ratios, in agreement with the previous studies of the sphere wake (Thompson et al, 2001).
The mode II Hopf transition was found to occur through a subcritical bifurcation from the steady, axisymmetric wake, to an unsteady three-dimensional wake.
The mode III transition was shown to be subcritical, and the subsequent Hopf transition over the same aspect ratio range was shown to be supercritical. This leads to the conclusion that the hairpin wake scales with the ring diameter, D, rather than the cross-section diameter, d, of the regular mode III transition, and therefore is the same transition mode for aspect ratios over the mode I and mode III transition range.

7. Acknowledgments

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